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A Three-Layer Supply Chain EPQ Model for Price- and Stock-Dependent Stochastic Demand with Imperfect Item Under Rework

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Abstract

In this paper, we have developed an integrated supplier-manufacturer-retailer, joint economic lot-sizing model for the items with stochastic demand and imperfect quality. The supplier produces the item (raw material) up to certain time, which is a decision variable, and sends it to the manufacturer. Now, the manufacturer produces the item in small cycles and the production process of manufacturer is imperfect which produces certain number of defective items. A 100 % screening process for detecting the imperfect quality items is conducted, and at the end of each cycle, the imperfect items are accumulated and are reworked by the manufacturer. Thus, ultimately the retailer receives the perfect quality item. We consider that the delivery quantity to the retailer depends on the price and stock-dependent stochastic demand of the retailer. The model considers the impact of business strategies such as optimal time, optimal ordering size of raw material, production rate, etc. in different sectors on collaborating marketing system. An analytical method is applied to optimize the production time and production rate to obtain minimum total cost. Finally, numerical results, which have several interesting managerial insights and implications, and the sensitivity analysis are presented and discussed for illustrative purposes.

Keywords: Supply chain model, Imperfect item, Price- and stock-dependent stochastic demand, Idle time

Introduction

Supply chain management has taken a very important and critical role for any company, with increasing globalization and competition in the market. A supply chain model (SCM) is a network of suppliers, producers (i.e., manufacturer), distributors (i.e., retailers), and customers which synchronizes a series of interrelated business processes. Nowadays, managers consider this type of modelling in order to obtain (1) optimum of raw materials from nature and then transporting it to a warehouse, (2) optimum production of goods in the production centre and distribution of these finished goods to retailers for sale to the customers. Thus, the researchers are focussing on the supply chain since the success of a firm may depend on its ability to link supply chain members consistently. Thus, integrated inventory management has recently received a great deal of attention. Goyal [12] considered the joint optimization problem of a single vendor and single buyer, in which he assumed that the vendor production rate is infinite. Goyal [13] extended the work by

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one vendor and multi-buyer integrated model where the shipment size increases geometrically. Hill [17] then generalized the model by considering the geometric growth factor as decision variable. Ramakrishna et al. [37] worked with a two-item and two-warehouse model with transhipment. Ben-Daya and Al-Nassar [3] worked on a three-layer supply chain integrated inventory production system. The work has been extended by Ben-Daya et al. [4]. Dellaert and Melo [9] worded on heuristic procedures for a stochastic lot-sizing problem in make-to-order manufacturing. Zhou and Guan [51] worked with two-stage stochastic lot-sizing problem under cost uncertainty. Liberopoulos et al. [27] worked with same type problem but for non-stop multi-grade production with sequence-restricted setup changeovers. Jha and Shanker [22] considered single vendor multiple buyer integrated production model with controlled lead time and service level constraints. The single-vendor multi-buyer integrated inventory supply chain in meeting deterministic demand has received a considerable attention of the researchers (namely, Lam and Ip [25], Hoque [18], Sarkar and Diponegoro [42], Zavanella and Zanoni [49, 50]; Srinivas and Rao [46], Solyal and Sural [44], Ben-Daya et al. [5], Jana et al. [21], Huang et al. [19], Pasandideh et al. [35], Pasandideh et al. [34], Lieckens and Vandaele [28], etc.).

The classical EPQ model assumes that the manufacturing process is failure free and all the items produced are of perfect quality throughout. However, in real production environment, it is observed that the defective items are produced due to imperfect production processes. Thus the inventory policy determined by the conventional model is inappropriate. So, the defective items must be rejected, repaired and reworked, and the corresponding substantial costs is incurred in the integrated total costing of the inventory system. Recently, numerous researchers are working on EPQ/EOQ models with imperfect quality items. This was triggered by Salameh and Jaber [38]. They developed an EOQ model to determine the optimal lot size where each lot delivered by the supplier contains imperfect items with a known probability density function. Hayek and Salameh [15] studied an EPQ model with the reworking of imperfect quality items. El-Kassar [10] examined an EOQ model with imperfect quality items, where the imperfect quality items are sold at a discounted price and the demands for both perfect and imperfect quality items are continuous during the inventory cycle. Chiu [8] considered an EPQ model with random defective rate, a reworking process, and backlogging. Liao and Sheu [26] described an integrated EPQ model with maintenance programs. An EPQ model with raw material of imperfect quality are used for production of finished item was studied by El-Kassar et al. [11]. Sana [39] investigated an EPL (economic production lot size) model in an imperfect production system in which the production facility could shift from an "in-control" state to an "out-of-control" state by and random time. Lin et al. [30] we propose an integrated supplier–retailer inventory model in which both supplier and retailer have adopted trade credit policies, and the retailer receives an arriving lot containing some defective items. Some recent works on inventory models with imperfect quality items is done by many researchers like Sana [40], Khan et al. [23], Soni and Patel [45], and Jaber et al. [20], etc.

Uncertainty plays a important role in most inventory management situations. The retailer wants enough supply to satisfy customer demands, but too much ordering increases holding costs and the risk of losses through obsolescence or spoilage. Also, too small order increases the risk of lost sales and unsatisfied customers. Thus the operations manager has to sets a master production schedule where he has to forecasts the imprecise

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nature of demands and optimize the quantity and manufacturing time to obtain maximum profit. Many work has been done by the researchers using deterministic condition. Pal et al. [33] worked with price and stock depended demand for deteriorating items. (Sarkar and Sarkar [43], Prasad and Mukherjee [36], etc.) worked on stock-dependent demand for deteriorating items and with partial backlogging. Thus in classical EOQ model the researchers works with deterministic approach and analyzed the inventory system which is not satisfactory when uncertainty is present. Nowadays researchers are working with uncertainty in demand, in production process and production time, in lead time, etc. Browne and Paul [6] concerned with the (r, q) inventory model, where demand accumulates continuously, but the demand rate at each instant is determined by an underlying stochastic process. Gumus and Guneri [14] worked on multi inventory stochastic and fuzzy supply chain. Manna et al. [31] worked with three layer supply chain model in fuzzy environment. Kumar et al. [24] worked on economic production lot size model with stochastic demand and shortage partial backlogging rate under imperfect quality items. Sana [41] developed an model for stochastic demand for limited capacity of own warehouse. Pal et al. [32] developed an EPQ inventory model to determine the optimal buffer inventory for stochastic demand in the market during preventive maintenance or repair of a manufacturing facility with an imperfect production system. Alshamrani [2] considers a stochastic optimal control of an inventory model with a deterministic rate of deteriorating items. Abdelsalam and Elassal [1] extended the work of Ben-Daya et al. [5] by relaxing the assumption of deterministic demand and constant holding cost and considering stochastic demand with varying ordering and holding cost. Lin and Wu [29] developed a model with combined pricing and supply chain operations under price-dependent Stochastic demand. He et al. [16] also worked on coordinating a supply chain with price dependent Stochastic demand. Now a days researchers (like Ting and Chung [47], Varyani et al. [48], Chen and Geunes [7], etc.) are considering various parameters as stochastic variable.

Nowadays, few researchers are paying focus on the effect of considering the idle time cost. Time management is extremely important in any business. This includes timing the completion of one project to coordinate with the beginning of another to reduce idle time. Idle time is the time associated with waiting, or when a piece of machinery is not being used but could be. To avoid any machinery or any other failure, sometime during the idle time, the parties (supplier and manufacture) repair and maintain the machineries and can train the worker. Also, too much of idle time means wastage of resources because machinery are idle and it is not in use. Thus, the manager has to optimize the time so that it does not effect the inventory system.

The purpose of this paper is to develop an integrated supplier-manufacturer-retailer production inventory model with imperfect product quality, price and stock-dependent stochastic demand. Deterioration is taken as constant and also the concept of idle time is taken under consideration. The imperfect items are reworked. The shipment of the item is done according to the need of the retailer which in turn depends upon the demand of the customer. The rest of the paper is organized with illustration of notations and assumptions used in the paper. Further the development of the mathematical formulation of this model has been depicted that integrates the supplier - manufacturer – retailer annual cost and takes under the consideration of imperfect production process and rework of the imperfect products. We have also constructed a numerical example using which the

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sensitivity analysis of various parameters are illustrated. Further we also have observed the supply demand relationship. Finally, we summarize and conclude the paper and provide directions for future research.

Assumptions

Notation:

P_s	Production rate for the supplier,
P_m	Production rate for the manufacturer,
P_1	Production rate of the reworked item (perfect unit)
D_{iR}	Stochastic demand of the retailer $i = 1, 2, 3, ...n$,
D_{iC}	Demand of the customer $i = 1, 2, 3, ...n + 1$.
$q_s(t)$	Inventory of the supplier
$q_{im}(t)$	Inventory of the manufacturer at each cycle, $i = 0, 1, 2, 3, ...n$,
$q_{iR}(t)$	Inventory of the retailer, received form the manufacturer in small batches, $i = 1, 2, 3, ...n$,
X	a continuous random variable,
x	Value of X,
Y	a continuous random variable, demand per season $Y = aq(t) - bs + X$, $a > b > 0$, where a, b are scale parameters,
y	Value of Y., i.e., $y = aq(t) - bs + x$,
Z	the percentage of defective units in the ordered lot which is a random variable
z	the value of Z
h_s	holding cost per unit per unit time for supplier ($/unit/time)
h_m	holding cost per unit per unit time for manufacturer ($/unit/time)
h_R	holding cost per unit per unit time for retailer ($/unit/time)
C_s	Purchase cost of unit item for supplier ($/unit)
C_m	Purchase cost of unit item for manufacturer ($/unit)
C_R	Purchase cost of unit item for retailer ($/unit)
A_s	Ordering cost for supplier
A_m	Ordering cost for manufacturer
A_R	Ordering cost for retailer
id_s	idle cost per unit time for supplier. ($/unit time)
id_m	idle cost per unit time for manufacturer ($/unit time)
id_R	idle cost per unit time for retailer ($/unit time)
s	unit selling price for the retailer,
n	number of cycle
r	number of cycle after which manufacturer stops receiving raw material.
t_s	Production time for supplier, which is a decision variable.
T_s	Cycle length for the supplier.
T_m	Cycle length for the manufacturer
T	Cycle length of the of the retailer.
T_{iR}	Time of each small batch for shipment from manufacturer to retailer, which in turn depends on the completion of inventory of retailer (variable quantity), where $T_{iR} = iT_R$, $i = 1, 2, 3, ..., n + 1$ where $T = (n + 1)T_R$.

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Assumptions:

1. Demand is price and stock-dependent stochastic, so demand per season is Y, i.e. $D_{iR} = D_{iC} = aq_{iR}(t) - bs + x$.

2. Let the probability density function f(x) of the demand x is,

$$f(x) = \lambda e^{-\lambda x} \quad where \quad 0 \leq x \leq \infty$$
$$= 0 \quad elsewhere$$

   which is a exponential distribution function where λ is the parameter of distribution.

3. Consider the probability density function g(z) for the rate of defective item z

$$g(z) = \frac{\theta}{1 - e^{-\theta}} e^{-\theta} \ where \ 0 \leq z \leq 1, \theta > 0$$
$$= 0 \quad elsewhere$$

   this is truncated exponential distribution where θ is the parameter of distribution.

4. Production rate of manufacturer is more than demand of manufacturer (because there are defective item, which are produced by the manufacturer, and are reworked and thereby finally good items are sent to retailer. So, to overcome the shortage from manufacturer end point more inventory is produced).

5. Lead time is negligible.

6. Inventory system is for a single item.

7. Defective item that are produced by the manufacturer are reworked and produced at the rate P_1.

8. Continuous shipment if quantity from supplier to manufacturer.

9. Idle time for supplier, manufacturer and retailer are also assumed. It is the time when the machine is idle and we have assumed that there is no failure in machinery at any point of time.

10. Their is no deterioration at retailer's end and hence no shortage at retailer end.

11. Item gets produced in the required time and supplier supplies continuously to manufacture which in turn supplies to retailer in batches and so it starts one cycle before the retailer.

12. At iT_R point of time (i.e., at regular interval of time), retailer determines the demand rate and orders that amount from the manufacturer, where $i = 1, 2, 3....n$.

13. To optimize the production cycle from manufacturer end we assume that until and unless sufficient quantity of the raw material is not available from supplier end then we will not run the cycle of manufacturer. Thus, for simplicity we assume that time when supplier end its inventory, manufacturer stops producing item and from that time onward manufacturer will only supply to the retailer as per their requirement.

Mathematical Model

We considered three stage joint economic lot sizing model for a supply chain problem with single supplier, single manufacturer and single retailer. The supplier produces and sends its finished goods to the manufacturer which is used as a raw material for the manufacturer. The manufacturer produces the item in small cycles which includes imperfect items also. At the end of each cycle the imperfect items are accumulated and it is reworked by the manufacturer. Thus, ultimately the retailer receives the perfect quality item. The

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demand of the retailer is price and stock depended stochastic demand which depends on the customer's choice. The amount of shipment from manufacturer to retailer depends on retailer's demand which in turn depends on customer's demand.

Formulation of Supplier Individual Cost

The integrated inventory model (Fig. 1) starts when $t = 0$ and stock is also zero. At that time, the supplier starts their production with the rate P_s unit per unit time and supply it to the manufacturer simultaneously. When $t = t_s$, suppliers stop their production, and at $t = T_s$, the inventory level of the supplier becomes zero. The total time of the integrated model is T, so the idle time for supplier is $T - T_s$.

$$\frac{dq_s}{dt} = \begin{cases} P_s - P_m & \text{for } 0 \leq t \leq t_s \\ -P_m & \text{for } t_s \leq t \leq T_s \\ 0 & \text{for } T_s \leq t \leq T \end{cases} \tag{1}$$

Along with the boundary conditions, $q_s(0) = 0$, $\quad q_s(t_s) = Q_s$, $\quad q_s(T_s) = 0$.

Thus, using boundary condition and solving equation no. (1) we get,

$$q_s(t) = \begin{cases} (P_s - P_m)t & \text{for } 0 \leq t \leq t_s \\ P_m(T_s - t) & \text{for } t_s \leq t \leq T_s \\ 0 & \text{for } T_s \leq t \leq T \end{cases} \tag{2}$$

Equating the Eq. (2) we get, time when supplier finishes it item $= T_s = \frac{kP_s}{P_m}t_s$ (where $k > 0$ is a scale parameter)

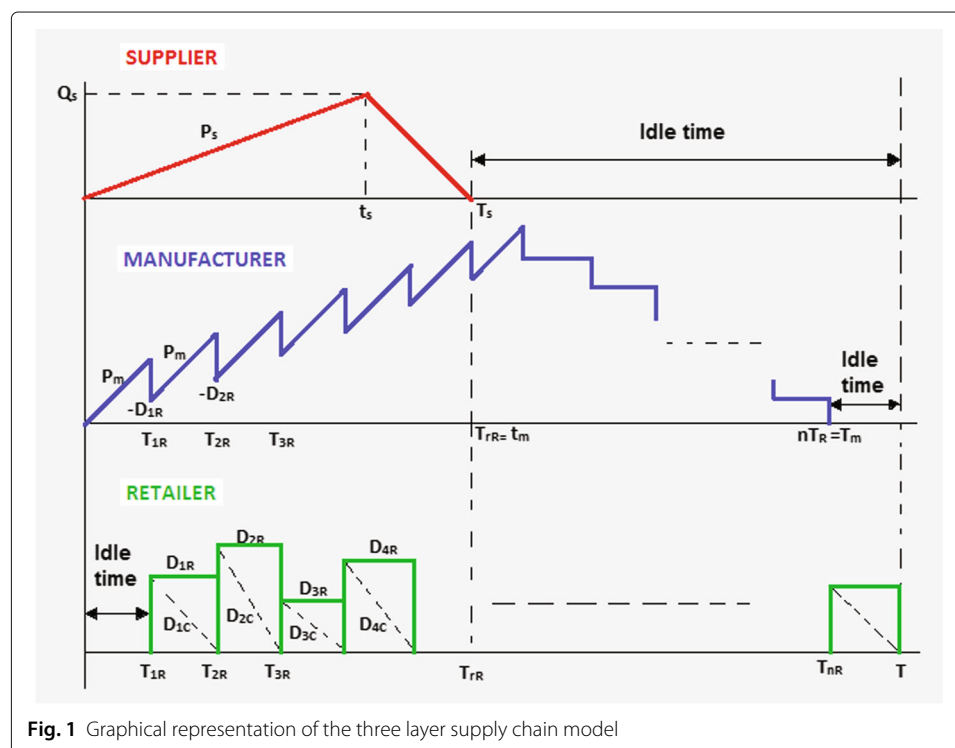Now since $kP_s t_s = P_m T_{rR} = rP_m T_R$, so, $T_R = \frac{kP_s}{P_m r}t_s$

Fig. 1 Graphical representation of the three layer supply chain model

Holding cost for the supplier (HC_s)

$$HC_s = h_s \left[\int_0^{t_s} (P_s - P_m)t\,dt + \int_{t_s}^{T_s} P_m(T_s - t)dt \right] = \frac{h_s}{2} \left[\frac{P_s^2 t_s^2}{P_m} - P_s t_s^2 \right]$$

Idle time for supplier $(IDT) = T - T_s = (n+1)T_R - T_s = \left(\frac{n+1}{r} - 1 \right) \frac{kP_s}{P_m} t_s$

Idle time cost$= (IDC_s) = id_s(IDT) = id_s \left(\frac{n+1}{r} - 1 \right) \frac{kP_s}{P_m} t_s$

Purchase cost of supplier$= (PC_s) = C_s P_s t_s$

Ordering cost $(OC_s) = A_s$

Therefore Total cost of manufacturer $= TC_s = HC_s + IDC_s + PC_s + OC_s$

$$= \frac{h_s}{2} \left[\frac{P_s^2 t_s^2}{P_m} - P_s t_s^2 \right] + id_s \left(\frac{n+1}{r} - 1 \right) \frac{kP_s}{P_m} t_s + C_s P_s t_s + A_s \tag{3}$$

Formulation of Manufacturer Individual Cost

The manufacturers start their production at the time $t = 0$ when the production rate P_m unit per unit time supply it to the retailer at the rate D_{iR} depending upon the demand of the customer D_{iC}. At the time $t = T_s = T_{rR}$, manufacturer stops receiving raw material from the supplier and thus stop producing the finished good. But, during $\left[T_{rR}, T_{(r+1)R} \right]$, the manufacturer rework on the imperfect item to make a perfect finished item. At the time $t = T_m = T_{nR}$, the stock of the manufacturer is zero. Thus, the idle time for the manufacturer is $T - nT_R$.

$$\frac{dq_m}{dt} = \begin{cases} (1-z)P_m, \ 0 \le t \le T_R & \text{where } q_m(0) = 0 \\ (1-z)P_m + zP_1, \ iT_R \le i \le (i+1)T_R & \text{where } i = 1, 2, 3, ..., r \\ \qquad\qquad\qquad \text{and } q_{im}(iT_R + 0) = q_{(i-1)m}(iT_R) - Di_R \\ zP_1, \ rT_R \le t \le (r+1)T_R, & \text{where } q_m(rT_R) = q_m(rT_R) - D_{rR} \end{cases} \tag{4}$$

And

$$q_{im}(t) = \begin{cases} q_{rm}[(r+1)T_R] - \sum_{K=r}^{i} D_{KR}, \ iT_R \le t \le (i+1)T_R \text{ where } i = r+1, ..., n-1 \\ \qquad\qquad\qquad\qquad\qquad\qquad (i.e., t_m \le t \le T_m) \\ \qquad\qquad 0 \qquad\qquad\qquad nT_R < t < T \end{cases} \tag{5}$$

Thus, using boundary condition and solving Eq. (4) and simplifying Eq. (5) we get,

$$q_m(t) = \begin{cases} (1-z)P_m t, & 0 \le t \le T_R, \text{ where } q_m(0) = 0 \\ (1-z)P_m t + zP_1(t - T_R) - \frac{T_R(x-bs)i(i+1)}{2} \left[1 + \frac{aT_R(2i+1)}{3} \right], \ iT_R \le i \le (i+1)T_R \text{ where } i = 1, 2, 3, ..., r \\ zP_1(t - T_R) + rT_R \left\{ (1-z)P_m - (x-bs) \left[\frac{r+1}{2} + \frac{aT_R(2r^2+3r+1)}{6} \right] \right\}, \ rT_R \le t \le (r+1)T_R, \\ T_R(x-bs)\frac{(i-r)}{6} \left[3(i+r+1) + aT_R \left[(i+r+1)^2 + i(i+1) + r(r+1) \right] \right], \ iT_R \le t \le (i+1)T_R \\ \qquad\qquad\qquad\qquad \text{where } i = r+1, ..., n-1 \ (i.e., t_m \le t \le T_m) \\ 0, & nT_R < t < (n+)T_R = T \end{cases}$$

Holding cost for manufacturer (HC_M) is

$$HC_M = h_m \left[\int_0^1 \left(\int_0^q \left\{ \int_0^{T_R} q_m(t)dt + \sum_{i=1}^{r-1} \int_{iT_R}^{(i+1)T_R} q_m(t)dt + \int_{rT_R}^{(r+1)T_R} q_m(t)dt \right.\right.\right.$$
$$\left.\left.\left. + \sum_{i=r+1}^{n} \int_{iT_R}^{(i+1)T_R} q_m(t)dt \right\} f(x)dx \right) g(z)dz \right]$$

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In the inventory system, the demand follows a special type of probabilistic density function f(x) as:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & where\ 0 \le x \le \infty \\ 0 & elsewhere \end{cases}$$

Also the rate of defective item follows the probability density function g(z) as:

$$g(z) = \begin{cases} \frac{\theta}{1-e^{-\theta}}\ e^{-\theta} & where\ 0 \le z \le 1,\ \theta > 0 \\ 0 & elsewhere \end{cases}$$

Thus Holding cost for manufacturer is

$$HC_M = \frac{h_m \theta}{1 - e^{-\theta}}\left[\left(\alpha + \frac{1}{\lambda} - \xi_1 - b\xi_2 s\right)\left(\frac{e^{-\lambda\beta-\theta} - e^{-\lambda\alpha}}{\alpha\lambda - \beta\lambda - \theta}\right) + \left(\frac{1}{\lambda} - \xi_1 - b\xi_2 s\right)\left(\frac{e^{-\theta} - 1}{\theta}\right)\right.$$
$$\left. + \xi_3\left(\frac{e^{-\theta}}{\theta} + \frac{e^{-\theta}}{\theta^2} - \frac{1}{\theta^2}\right) - (\xi_3 + \beta - \alpha)\left(\frac{e^{-\lambda\beta-\theta}}{\alpha\lambda - \beta\lambda - \theta} + \frac{e^{-\lambda\beta-\theta} - e^{-\alpha\lambda}}{(\alpha\lambda - \beta\lambda - \theta)^2}\right)\right]$$

where

$$q = (1 - z)P_m t_m + zP_1 T_{(r+1)R}\ where\ t_m = T_{rR}.$$

$$\alpha = P_m t_m = P_m r T_R$$

$$\beta = (r + 1)P_1 T_R$$

$$\xi_1 = \frac{P_m T_R^2}{2} + \frac{P_m T_R^2}{2}(r^2 - 1) + rP_m T_R^2 + (n - r)rT_R P_m$$

$$\xi_2 = \left[\frac{(r^2 - 1)(arT_R + 2)T_R}{12} + \frac{(r + 1)T_R}{2} + \frac{aT_R^2(2r + 1)(r + 1)}{6} + \frac{(r + 1)r(n - r)}{2}\right.$$
$$\left. \times \left(1 + \frac{aT_R(2r + 1)}{3}\right) + \xi_4\right]T_R$$

$$\xi_3 = \left[\xi_1 - \frac{P_1 T_R^2 r^2}{2} - (n - r)rP_1 T_R\right]$$

$$\xi_4 = \frac{1}{6}\left[aT_R\left\{\frac{n^2(n + 1)^2}{2} - \frac{(r + 1)^2(r + 2)^2}{2}\right\} + (3 + r - arT_R)\right.$$
$$\times \left[\frac{n(n + 1)(2n + 1)}{3} - \frac{(r + 1)(r + 2)(2r + 3)}{3}\right]$$

$$+ [(r + 1)(3 + aT_R(2r + 1)) - 2r(r + 3)]\left[\frac{n(n + 1)}{2} - \frac{(r + 1)(r + 2)}{2}\right]$$

$$\left. - r(n - r)(r + 1)(3 + aT_R(2r + 1))\right]$$

Total idle cost $(IDC_M) = id_m(T - nT_R) = id_m T_R = id_m \frac{kP_s}{P_m}t_s$

Purchase cost $(PC_M) = C_m P_m T_S = C_m kP_s t_s$

Ordering cost $(OC_M) = A_m$

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Therefore, total cost of manufacturer

$$
\begin{aligned}
TC_M &= HC_M + IDC_M + PC_M + OC_M \\
&= \frac{h_m \theta}{1 - e^{-\theta}} \left[\left(\alpha + \frac{1}{\lambda} - \xi_1 - b\xi_2 s \right) \left(\frac{e^{-\lambda\beta-\theta} - e^{-\lambda\alpha}}{\alpha\lambda - \beta\lambda - \theta} \right) + \left(\frac{1}{\lambda} - \xi_1 - b\xi_2 s \right) \right. \\
&\quad \times \left. \left(\frac{e^{-\theta} - 1}{\theta} \right) + \xi_3 \left(\frac{e^{-\theta}}{\theta} + \frac{e^{-\theta}}{\theta^2} - \frac{1}{\theta^2} \right) \right. \\
&\quad \left. -(\xi_3 + \beta - \alpha) \left(\frac{e^{-\lambda\beta-\theta}}{\alpha\lambda - \beta\lambda - \theta} + \frac{e^{-\lambda\beta-\theta} - e^{-\alpha\lambda}}{(\alpha\lambda - \beta\lambda - \theta)^2} \right) \right]
\end{aligned}
\tag{6}
$$

$$
+ id_m \frac{kP_s}{P_m} t_s + C_s P_m k T_S + A_m
\tag{7}
$$

where $\xi_1, \xi_2, \xi_3, \alpha, \beta$ are mentioned above.

Formulation of Retailer Individual Cost

The retailer starts selling this product to the customer at the time $t = T_R$ and end selling at $t = (n + 1)T_R$ according to the demand of the customer. The idle period for retailer is $[0, T_R]$.

$$
\frac{dq_{iR}(t)}{dt} = -D_{iC} = -D_{iR}, \quad i = 1, 2, 3, ..., n, \quad \text{where } q_{iR}(T_{iR}) = D_{iR} T_{iR}
\tag{8}
$$

Thus thus using boundary condition and solving and simplifying Eq. (8), we get

$$
q_{iR}(t) = (x - bs) \left[\frac{2T_{iR} - t}{1 - aT_{iR}} \right], \quad i = 1, 2, 3, ..., n
\tag{9}
$$

Holding cost of retailer

$$
\begin{aligned}
HC_R &= h_R \left[\int_0^1 \left(\int_0^q \left\{ \sum_{i=1}^{n} \int_{iT_R}^{(i+1)T_R} q_{iR}(t) dt \right\} f(x) d(x) \right) g(z) dz \right] \\
&= \frac{h_R \theta \eta_1}{1 - e^{-\theta}} \left[\left(\frac{1}{\lambda^2} - \frac{bs}{\lambda} \right) \left(\frac{e^{-\theta} - 1}{\theta} - \frac{e^{-\lambda\beta-\theta} - e^{-\lambda\alpha}}{\alpha\lambda - \beta\lambda - \theta} \right) - \frac{\beta e^{-\lambda\beta-\theta} - \alpha e^{-\lambda\alpha}}{\lambda \left(\alpha\lambda - \beta\lambda - \theta \right)} \right. \\
&\quad \left. - \frac{(\alpha - \beta) \left(e^{-\lambda\beta-\theta} - e^{-\lambda\alpha} \right)}{(\alpha\lambda - \beta\lambda - \theta)^2} \right]
\end{aligned}
$$

where $\eta_1 = \left(\frac{T_R^2 n^2}{2} - \frac{aT_R^3 n(n^2-1)}{3} \right) \lambda$ and $f(x)$ and $g(z)$ is define above.

Total idle cost $(IDC_R) = id_R T_R$

Since retailer receives perfect quality from the manufacturer, thus the retailer has to purchase.

Purchase cost $(PC_R) = C_R \left[\int_0^1 \left(\int_0^q \left\{ \sum_{i=1}^{n} q_{iR}(T_{iR}) \right\} f(x) d(x) \right) g(z) dz \right] =$

$$
\begin{aligned}
&= \frac{c_R T_R \theta n(n + 1)}{2(1 - e^{-\theta})} \left(1 + \frac{aT_R(2n + 1)}{3} \right) \left[\frac{\lambda\alpha - 1}{\lambda(\lambda\alpha - \lambda\beta - \theta)} (e^{-\lambda\beta-\theta} - e^{-\alpha\lambda}) - \frac{1}{\theta\lambda} (e^{-\theta} - 1) \right. \\
&\quad \left. -(\alpha - \beta) \left[e^{-\lambda\beta-\theta} - \frac{(e^{-\lambda\beta-\theta} - e^{-\alpha\lambda})}{\lambda(\lambda\alpha - \lambda\beta - \theta)^2} \right] \right]
\end{aligned}
$$

Ordering cost $(OC_r) = A_R$

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Therefore, total cost of retailer $TC_R = HC_R + IDC_R + PC_R + OC_R$

$$= \frac{h_R \theta \eta_1}{1 - e^{-\theta}} \left[\left(\frac{1}{\lambda^2} - \frac{bs}{\lambda} \right) \left(\frac{e^{-\theta} - 1}{\theta} - \frac{e^{-\lambda\beta-\theta} - e^{-\lambda\alpha}}{\alpha\lambda - \beta\lambda - \theta} \right) - \frac{\beta e^{-\lambda\beta-\theta} - \alpha e^{-\lambda\alpha}}{\lambda(\alpha\lambda - \beta\lambda - \theta)} \right. \tag{10}$$

$$\left. - \frac{(\alpha - \beta)\left(e^{-\lambda\beta-\theta} - e^{-\lambda\alpha}\right)}{(\alpha\lambda - \beta\lambda - \theta)^2} \right]$$

$$+ id_R T_R + \frac{c_R T_R \theta n(n+1)}{2(1 - e^{-\theta})} \left(1 + \frac{a T_R (2n+1)}{3} \right) \left[\frac{\lambda\alpha - 1}{\lambda(\lambda\alpha - \lambda\beta - \theta)} (e^{-\lambda\beta-\theta} - e^{-\alpha\lambda}) \right.$$

$$\left. - \frac{1}{\theta\lambda}(e^{-\theta} - 1) - (\alpha - \beta) \left[e^{-\lambda\beta-\theta} - \frac{(e^{-\lambda\beta-\theta} - e^{-\alpha\lambda})}{\lambda(\lambda\alpha - \lambda\beta - \theta)^2} \right] \right] + A_R \tag{11}$$

Therefore, the total costing of the inventory is $TC = TC_s + TC_M + TC_R$.

We have considered two cases: case 1 we optimize the total cost with respect to production cycle time of supplier(t_s) while in case 2 we optimize the total cost with respect to production rate of the supplier (P_s).

Case 1 *Production cycle time of supplier (t_s) is the decision variable.*

Lemma 1 *$TC(t_s)$ has a global minimum for $t_s \in [0, \infty)$ provided all three conditions are satisfied*

(i) $P_s > P_m$

(ii) $rn^2 T_s + \frac{arC_r n(1+n)(1+2n)P_s^2 \theta \sigma}{3(1-e^{-\theta})P_m^2} > 2an(n^2-1)P_s t_s \varphi$

(iii) $\frac{d^2 TC_m}{dt_s^2} > 0$

Proof As the total cost TC is function of t_s so we have to minimize the total cost and obtain the optimum value of the production time for supplier taken is t_s^*. If $\frac{dTC}{dt_s}$ exits for $t_s \in [0, \infty)$ then the necessary condition for $TC(t_s)$ to be minimized is $\frac{dTC}{dt_s} = 0$ and thus obtain the point $t_s = t_s^*$. Thus the optimal value of t_s^* is such that $TC(t_s)$ has the minimum value if the derived value $TC^*(t_s^*)$ must satisfy the sufficient condition, $\frac{d^2 TC}{dt_s^2} |_{t_s = t_s^*} > 0$

Now, let us check the sufficient condition: $\frac{d^2 TC}{dt_s^2} = \frac{d^2 TC_s}{dt_s^2} + \frac{d^2 TC_m}{dt_s^2} + \frac{d^2 TC_R}{dt_s^2}$ at the point $t_s = t_s^* \frac{d^2 TC_s}{dt_s^2} = h_s \left[\frac{P_s^2}{P_m} - P_s \right] > 0$ (from condition (i) i.e., $P_s > P_m$) $\frac{d^2 TC_R}{dt_s^2} = \frac{n^2 P_s^2}{P_m^2 r^2} + \frac{a C_r n(1+n)(1+2n)P_s^4 \theta \sigma}{3(1-e^{-\theta})P_m^4 r^2} - \frac{2an(n^2-1)P_s^3 t_s \lambda \varphi}{P_m^3 r^3} > 0$ (from condition (ii) and $\frac{d^2 TC_m}{dt_s^2} > 0$

Thus,

$$\frac{d^2 TC}{dt_s^2} = h_s \left[\frac{P_s^2}{P_m} - P_s \right] + \frac{h_m \theta}{1 - e^{-\theta}} \left[\left(-\frac{d^2 \xi_1}{dt_s^2} - bs \frac{d^2 \xi_2}{dt_s^2} \right) \left(\frac{e^{-\lambda\beta-\theta} - e^{-\lambda\alpha}}{\alpha\lambda - \beta\lambda - \theta} \right) \right.$$

$$+ \left(-\frac{d^2 \xi_1}{dt_s^2} - bs \frac{d^2 \xi_2}{dt_s^2} \right) \left(\frac{e^{-\theta} - 1}{\theta} \right) + \frac{d^2 \xi_3}{dt_s^2} \left(\frac{e^{-\theta}}{\theta} + \frac{e^{-\theta}}{\theta^2} - \frac{1}{\theta^2} \right)$$

$$\left. - \frac{d^2 \xi_3}{dt_s^2} \left(\frac{e^{-\lambda\beta-\theta}}{\alpha\lambda - \beta\lambda - \theta} + \frac{e^{-\lambda\beta-\theta} - e^{-\alpha\lambda}}{(\alpha\lambda - \beta\lambda - \theta)^2} \right) \right] + \frac{n^2 Ps^2}{P_m^2 r^2}$$

$$+ \frac{a C_r n(1+n)(1+2n)P_s^4 \theta \sigma}{3(1 - e^{-\theta})P_m^4 r^2} - \frac{2an(n^2-1)P_s^3 t_s \lambda \varphi}{P_m^3 r^3} > 0$$

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Where, $\sigma = \frac{\lambda\alpha-1}{\lambda(\lambda\alpha-\lambda\beta-\theta)}(e^{-\lambda\beta-\theta}-e^{-\alpha\lambda})-\frac{1}{\theta\lambda}(e^{-\theta}-1)-(\alpha-\beta)\left[e^{-\lambda\beta-\theta}-\frac{(e^{-\lambda\beta-\theta}-e^{-\alpha\lambda})}{\lambda(\lambda\alpha-\lambda\beta-\theta)^2}\right]$

$$\varphi = \frac{h_R\theta}{1-e^{-\theta}}\left[\left(\frac{1}{\lambda^2}-\frac{bs}{\lambda}\right)\left(\frac{e^{-\theta}-1}{\theta}-\frac{e^{-\lambda\beta-\theta}-e^{-\lambda\alpha}}{\alpha\lambda-\beta\lambda-\theta}\right)-\frac{\beta e^{-\lambda\beta-\theta}-\alpha e^{-\lambda\alpha}}{\lambda(\alpha\lambda-\beta\lambda-\theta)}\right.$$
$$\left.-\frac{(\alpha-\beta)\left(e^{-\lambda\beta-\theta}-e^{-\lambda\alpha}\right)}{(\alpha\lambda-\beta\lambda-\theta)^2}\right]$$

$$\frac{d^2\xi_1}{dt_s^2} = \frac{P_s^2}{P_mr^2}+\frac{2P_s^2}{P_mr}+\frac{P_s^2(r^2-1)}{P_mr^2}$$

$$\frac{d^2\xi_2}{dt_s^2} = -\frac{P_1P_s^2}{P_m^2}+\frac{P_s^2}{P_mr^2}+\frac{2P_s^2}{P_mr}+\frac{P_s^2(r^2-1)}{P_mr^2}$$

$$\frac{d^2\xi_3}{dt_s^2} = T_R\left[\frac{aP_s^2(r+1)(2r+1)}{3P_m^2r^2}+\frac{aP_s^2(r^2-1)}{6P_m^2r}\right]+\frac{2P_s}{P_mr}\left[\frac{P_s(r+1)}{2P_mr}\right.$$
$$+\frac{aP_s(n-r)(r+1)(2r+1)}{6P_m}+\frac{1}{6}\left(-\frac{aP_s(n-r)(r+1)(2r+1)}{P_m}\right.$$
$$+\frac{aP_s(r+1)(2r+1)\left(\frac{1}{2}n(n+1)-\frac{1}{2}(r+1)(r+2)\right)}{P_mr}$$
$$+\frac{aP_s\left(\frac{1}{4}n^2(n+1)^2-\frac{1}{4}(r+1)^2(r+2)^2\right)}{P_mr}$$
$$\left.-\frac{aP_s\left(\frac{1}{3}n(n+1)(2n+1)-\frac{1}{3}(r+1)(2r+3)(r+2)\right)}{P_m}\right)$$
$$\left.+\frac{aP_s^2(r+1)(2r+1)t_s}{3P_m^2r^2}+\frac{aP_s^2(r^2-1)t_s}{12P_m^2r}+\frac{P_s(r^2-1)(2+\frac{aP_st_s}{P_m})}{12P_mr}\right]$$

This ensures that the objective function TC is minimized for $t_s = t_s^*$. $\qquad\square$

***Case* 2.** *Production rate of the supplier* (P_s) *as decision variable.*

In three-layer supply chain model, production rate of supplier is one of the most important deciding factor for inventory control. A situation in which the demand decreases (or increases) may cause the manufacturers and suppliers to decrease (or increase) their production as well. Also, the production rate may either increase or decrease with the inventory level. If the production rate of supplier increases more than the demand then it will tend the hold the inventory and increase the holding cost of the supplier. Again by producing less than the demand the supplier will deliver the raw materials with a delay to manufacturer who will have to plan his inventory accordingly to optimize his cost and reduce shortages. Therefore, in this case, we have tried to optimize the total cost of inventory of supplier, manufacturer and retailer with supplier production rate as decision variable. The problem that is interesting in this case is the production planning problem. We consider supplier produces a single product which is sold immediately to the manufacturer. The problem is presented as an optimal control problem with control variable (production rate). Typically, the firm has to balance these costs and find the quantity which should produce in order to keep the total cost minimum.

As the total cost TC is function of P_s so we have to minimize the total cost and obtain the optimum value of the production rate for supplier taken is t_s^*. The necessary condition for $TC(P_s)$ to be minimized is $\frac{dTC}{dP_s}=0$ and thus obtain the point $P_s = P_s^*$. To ensure that

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the objective function is convex for the optimal value of P_s^* is such that $TC(P_s)$ has the minimum value if the derived value $TC^*(P_s^*)$ must satisfy the sufficient condition,

$$\frac{d^2 TC}{dP_s^2}\Big|_{P_s=P_s^*} > 0. \tag{12}$$

Here, we have optimize the production rate of supplier to know that at what rate the supplier should produce so that the total cost is minimized and there is no shortage at manufacturer and retailer end.

Since TC is a very complicated function, with high powers in the expression, it is impossible to show the analytical validity of the above sufficient condition. Thus the inequality Eq. (12) is assessed and shown numerically.

Numerical Analysis

The production of high variety products available in supermarket with short life cycles, such as computer parts, fashion clothes, some food items and many others, has remarkably pushed different companies towards high levels of competition. To sustain in the competitive market, firms can no longer operate as individual and autonomous entities. Hence, the companies realizes the necessity of having mutual understanding and better collaboration with their suppliers, manufacturers, retailers, and customers.

So, in this paper, we have considered three layer supply chain model where suppliers, manufacturer, and retailer plan, implement and manage the flow of inventory to optimize the total cost of the inventory. Let us consider the given numerical example as below.

Example 1 *In a supermarket let the retailer has price and stock dependent stochastic demand $D_R = aq_{iR}(t) - bs + x$ (where $a = 0.8$, $b = 5$, $s = 30$ and x is continuous random variable) based on which it orders the required amount of quantity in batches from the manufacturer considering that there is no shortages. The retailer purchase the finished good from the manufacturer at the cost of $20\$$ per unit item and hold it at the cost of $2.1\$$ per unit item. In order to set up infrastructure and to maintain the inventory the retailer spend $5200\$$ per unit item. Initially when there was no product with the retailer it remains idle, which costs $4\$$ per unit time.*

Estimating the order from retailer, the manufacturer produces the required item at the rate of 0.1 units per unit time and the imperfect items are reworked at the rate of 0.2 units per unit time. The manufacturer produces the item at upto 6 cycle and in the 7th cycle the manufacturer reworked the imperfect item which is produced in the 6th cycle and remaining item in the inventory is finished by 9th cycle. The manufacturer purchases the raw material from the supplier at the cost of $15\$$ per unit item and holds it at the cost of $1.5\$$ per unit item. In order to set up infrastructure and to maintain the inventory the manufacturer spend $4000\$$ per unit item. At the 10th cycle (i.e., end of manufacturer cycle) when there is no product with the manufacturer, it remains idle which costs $3\$$ per unit time.

The supplier based on manufacturer requirement produces the raw material at the rate of 0.15 units per unit time. The supplier purchase the raw material at the cost of $12\$$ per unit item and hold it at the cost of $1\$$ per unit item. In order to set up infrastructure and to maintain the inventory the manufacturer spend $3500\$$ per unit item. At the end of supplier's inventory cycle (i.e., there is no product with the supplier) it remains idle which costs $2\$$ per unit time.

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Data considered to illustrate the model is as below:

$C_s = 12, \ C_m = 15, \ C_r = 20, \ h_s = 0.7, \ h_m = 1.5, \ h_R = 2.1, \ id_s = 2, \ id_m = 3,$

$id_R = 4, \ s = 30, \ P_s = 0.15, \ P_m = 0.1, \ P_1 = 0.2, \ n = 9, \ r = 6, \ a = 0.8, \ b = 5,$

$\theta = 0.1, \ \lambda = 0.25, \ A_s = 3500,$

$A_m = 4000, \ A_R = 5200,$ for simplicity let us assume $k = 1$.

Case 1. We optimize the total cost with respect to production cycle of supplier (t_s) (Fig. 2). Thus, we get the optimum value $T_R^* = 0.352, \ t_s^* = 1.407, \ t_m^* = T_s^* = 2.111, \ T = 3.518,$ and $TC^* = 10700.5\$$

Sensitivity Analysis

Sensitivity analysis is done to check the percentage change in the final cost by changing any one of the parameter by $-20\%, -10\%, 10\%, 20\%$ and keeping the other parameter fixed. Since n and r are integer numbers, thus we have to check the sensitivity of the parameter by changing the number of cycle by $-2, -1, 1, 2$.

From Table 1 and Fig. 3, we can observe that total number of cycle (n) is highly sensitive. As n increases the total cost decreases and vice versa. This is because as the no of cycles increases, the time for each batch delivery decreases. Manufacturer do not have to hold the inventory of items for long and for retailer due to greater availability of stock it increases the demand and which in turn decreases its holding inventory. The result of decrease in holding cost decreases the total cost and vice versa.

While for a fixed total no. of cycle (n) and $n > r$, if the no. of production cycle (r) decreases the total cost also decreases and it is highly sensitive. This is because for less number of production cycle the manufacturer has to produce more amount in individual cycle and it require more time to prepare the lot amount hence the total cycle time also increases. As more amount of items are produced due to the longer production run time, the rework cost, labor cost, energy cost and other costs per unit product decreases automatically and thus the total costing decreases and vice versa.

From the above Table 2 and Fig. 4, we observe that selling price (s), Retailer's purchase cost (C_r), Holding cost (h_R) are highly sensitive, Holding cost of the manufacturer (h_m) is
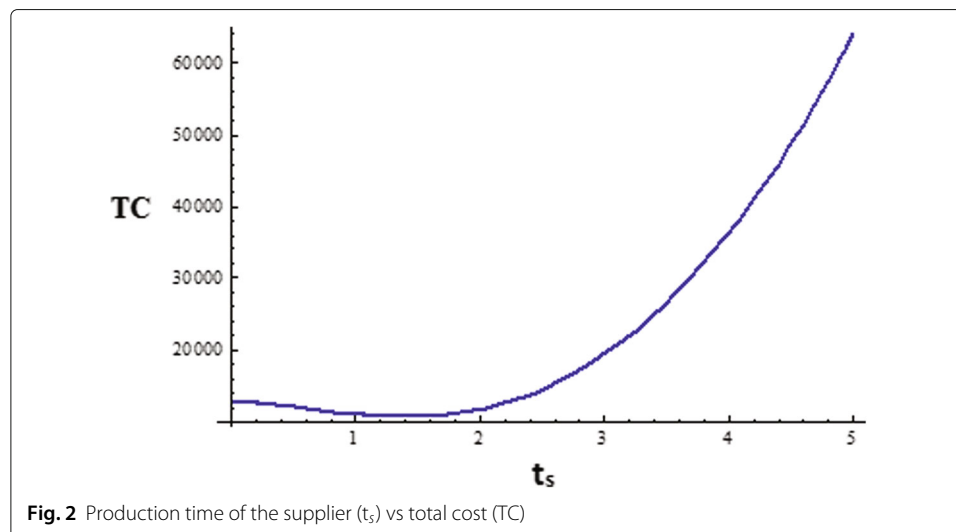

Fig. 2 Production time of the supplier (t_s) vs total cost (TC)

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Table 1 Variation of n and r with respect to TC

Paramter	change	T_R^* (year)	t_s^* (year)	$t_m^* = T_s^*$ (year)	T^* (year)	TC*	% change of TC
	−2	0.311	1.242	1.863	2.484	12050.2	12.61
n	−1	0.32	1.328	1.92	2.88	11509.4	7.56
	+1	0.369	1.477	2.216	4.062	9566.63	−10.6
	+2	0.385	1.538	2.307	4.614	8053.92	−24.73
	−2	0.756	2.017	3.026	7.564	1933.68	−81.93
r	−1	0.481	1.603	2.405	4.809	8697.96	−18.71
	+1	0.289	1.348	2.022	2.889	11419.7	6.72
	+2	0.265	1.412	2.118	2.648	11666.8	9.03

moderately sensitive, and finally purchase cost of the supplier and manufacturer (C_s and C_m), idle cost of the supplier, manufacturer and retailer (id_s, id_m, and id_R) are less sensitive. Thus, we can see that the sensitivity of various cost for the supplier and manufacturer is very less and that of retailer is very high.

(i) We observe that as selling price (s) increases, the total cost decreases. This is because the demand is price and stock dependent so if the price increases, the demand decreases which results in increase in the total cost and vice versa. We observe that this parameter is highly sensitive.

(ii) If we decrease the purchase cost (C_r) of the retailer then the total cost decreases and vice versa and it is highly sensitive , but when purchase cost (C_r) of the retailer increases the total cost increases moderately. If purchase cost (C_r) of the retailer increases means cost of purchasing the raw material increases, so then the total cost of the retailer also increases which in turn increases the total cost of the inventory. The same is true for supplier and manufacturer but those parameters are less sensitive. That is if the purchase cost of the supplier and manufacturer (C_s and C_m) increases means cost of purchasing the raw material increases, so the total cost of the supplier and manufacturer also increases respectively, which in turn increases the total cost of the inventory.

(iii) We observe that holding cost of supplier is insensitive while that of manufacturer is less sensitive and of retailer is highly sensitive. From the table we note that if the
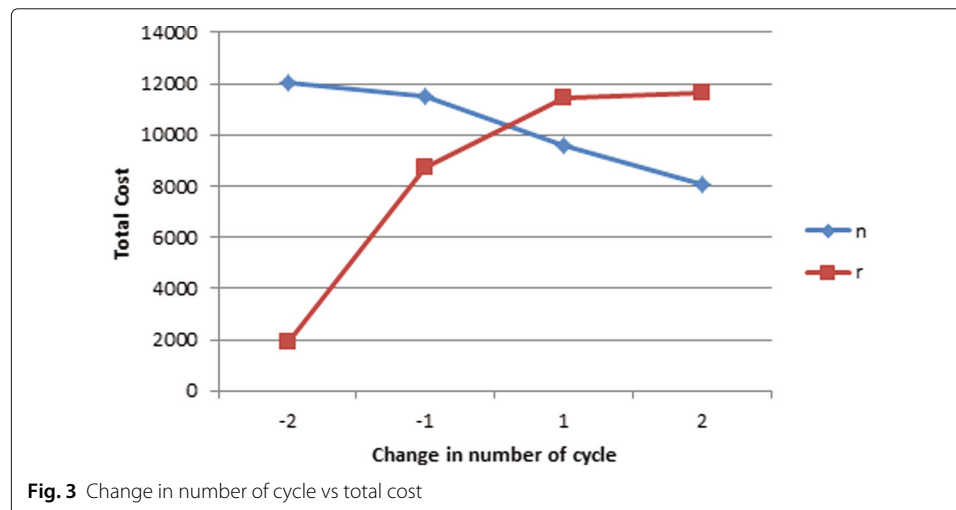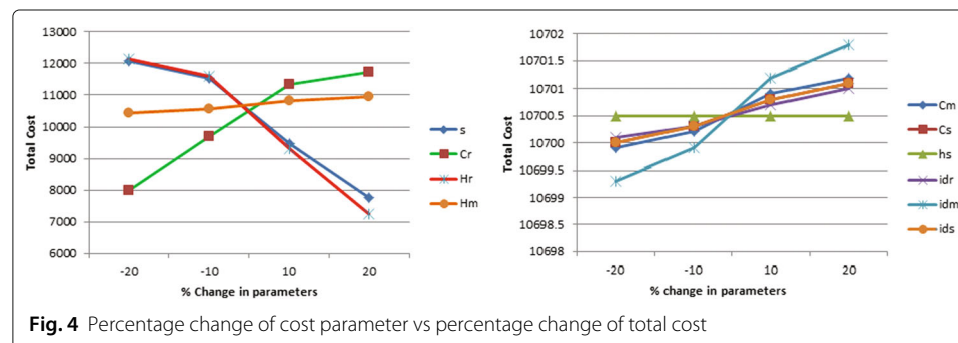

Fig. 3 Change in number of cycle vs total cost

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Table 2 Sensitivity analysis for cost parameters

Paramter	% change	T_R^* (year)	t_s^* (year)	$t_m^* = T_s^*$ (year)	T^* (year)	TC^*	% change of TC
	−20	0.226	0.902	1.353	2.255	12077.3	12.87
s	−10	0.286	1.142	1.713	2.855	11534.7	7.79
	10	0.425	1.699	2.549	4.248	9478.1	−11.42
	20	0.505	2.021	3.032	5.053	7750.16	−27.57
	−20	0.542	2.167	3.251	5.418	8001.65	−25.22
C_R	−10	0.431	1.723	2.585	4.308	9696.81	−9.38
	10	0.293	1.172	1.758	2.93	11328	5.86
	20	0.248	0.991	1.487	2.478	11737.2	9.69
	−20	0.35181	1.40722	2.11083	3.5181	10699.9	−0.006
C_m	−10	0.35179	1.40717	2.11076	3.5179	10700.2	−0.003
	10	0.35177	1.40707	2.11062	3.5177	10700.9	0.004
	20	0.35176	1.40704	2.11056	3.516	10701.2	0.007
	−20	0.3518	1.4072	2.1108	3.518	10700	−0.005
C_s	−10	0.35179	1.40716	2.11074	3.5179	10700.3	−0.002
	10	0.35177	1.40709	2.11063	3.51772	10700.8	0.003
	20	0.35176	1.40705	2.11057	3.51762	10701.1	0.006
	−20	0.214	0.857	1.286	2.143	12148.6	13.53
h_R	−10	0.279	1.115	1.673	2.788	11596.7	8.38
	10	0.435	1.740	2.610	4.350	9316.04	−12.94
	20	0.531	2.123	3.185	5.308	7248.56	−32.26
	−20	0.373	1.492	2.238	3.730	10421.3	−2.61
h_m	−10	0.362	1.449	2.174	3.623	10565.4	−1.26
	10	0.342	1.367	2.051	3.418	10827.2	1.18
	20	0.332	1.328	1.992	3.32	10946.1	2.3
	−20	0.352	1.4072	2.1108	3.518	10700.1	−0.004
id_r	−10	0.352	1.4072	2.1108	3.518	10700.3	−0.002
	10	0.35178	1.4071	2.11065	3.5178	10700.7	0.002
	20	0.35178	1.4071	2.11065	3.5178	10701	0.005
	−20	0.35183	1.4073	2.11095	3.5183	10699.3	−0.011
id_m	−10	0.352	1.4072	2.1108	3.518	10699.9	−0.006
	10	0.35175	1.407	2.1105	3.5175	10701.2	0.007
	20	0.35173	1.4069	2.11035	3.5173	10701.8	0.012
	−20	0.352	1.4072	2.1108	3.518	10700	−0.005
id_r	−10	0.352	1.4072	2.1108	3.518	10700.3	−0.002
	10	0.35178	1.4071	2.11065	3.5178	10700.8	0.003
	20	0.35175	1.407	2.1105	3.5175	10701.1	0.006

Fig. 4 Percentage change of cost parameter vs percentage change of total cost

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holding cost of manufacturer increases then the total cost also increases. This is true in reality because the manufacturer has to hold the item which it produces and thus the total costing of manufacturer increases and hence the total inventory cost increases. But, if the holding cost of retailer increases then the total cost also decreases. This is because the demand of the retailer is price and stock dependent and so to optimize the retailer try to keep less stock thus the total cost of the retailer decreases which in turn decreases the total costing of the inventory.
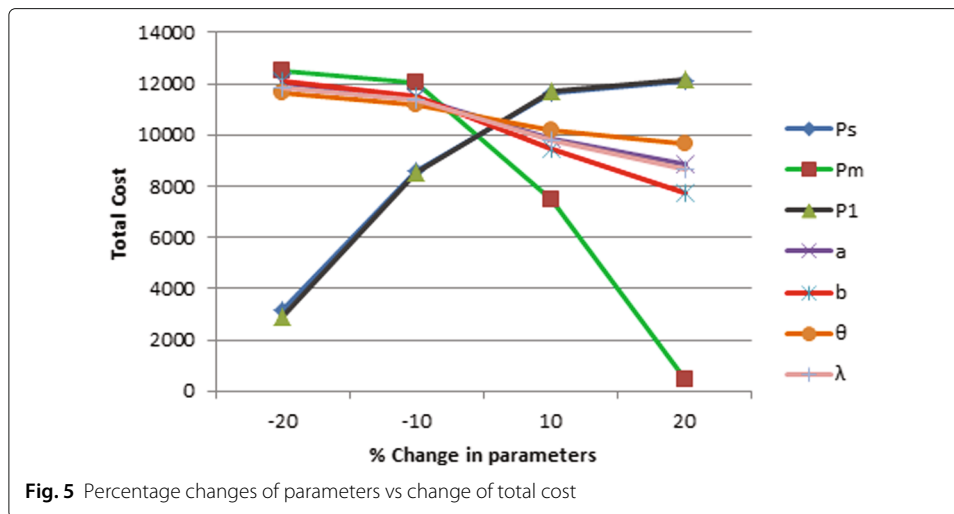
(iv) The idle cost of the supplier, manufacturer and retailer are very less sensitive. This has positive effect in our model since idle time means wastage of resources because machinery are idle and it is not in use. So, this means we can use this time for repairing machinery default, if any, by not effecting to inventory costing.

From Table 3 and Fig. 5, we observe that production rate of supplier (P_s), manufacturer (P_m) and the rework rate of the manufacturer (P_1), and demand factor (b) are highly sensitive. The demand factor (a), parameter (θ), and (λ) are moderately sensitive.

(i) We observe that as the production rate of the supplier (P_s) increases the total cost of the inventory also increases and vice versa. This is true because supplier will produce more item in short time and thus the overall costing increases also once the item is produced it is transferred to the manufacturer where latter has to hold the item for longer time. Thus, considering all the effects, the total inventory cost increases. The

Table 3 Sensitivity analysis for various parameters

Paramter	% change	T_R^* (year)	t_s^* (year)	$t_m^* = T_S^*$ (year)	T^* (year)	TC^*	% change of TC
	−20	0.806	4.032	4.838	8.06	3170.33	−70.37
P_s	−10	0.513	2.282	3.081	5.135	8583.53	−19.78
	10	0.255	0.926	1.528	2.54	11641.7	8.79
	20	0.192	0.6397	1.151	1.919	12104.5	13.12
	−20	0.118	0.378	0.709	1.181	12505.6	16.87
P_m	−10	0.21	0.755	1.258	2.097	12020.8	13.34
	10	0.564	2.483	3.386	5.643	7470.6	−30.18
	20	0.859	4.123	5.154	8.589	435.84	−95.93
	−20	0.712	2.85	4.275	7.125	2904.13	−72.86
P_1	−10	0.486	1.944	2.916	4.86	8529.98	−20.28
	10	0.263	1.052	1.578	2.63	11684.7	9.2
	20	0.199	0.799	1.199	1.998	12173.6	13.77
	−20	0.255	1.018	1.527	2.545	12030.2	12.43
a	−10	0.307	1.229	1.844	3.073	11443.9	6.95
	10	0.3898	1.559	2.339	3.898	9822.67	−8.2
	20	0.423	1.690	2.535	4.225	8830.38	−17.48
	−20	0.226	0.902	1.353	2.255	12077.3	12.87
b	−10	0.286	1.142	1.713	2.855	11534.7	7.79
	10	0.425	1.699	2.549	4.248	9478.1	−11.42
	20	0.505	2.021	3.032	5.053	7750.16	−27.57
	−20	0.272	1.086	1.629	2.715	11608.4	8.48
θ	−10	0.313	1.251	1.877	3.128	11179.1	4.47
	10	0.389	1.556	2.334	3.89	10180	−4.86
	20	0.424	1.697	2.546	4.243	9623.54	−10.06
	−20	0.226	0.904	1.356	2.26	11862.2	10.86
λ	−10	0.285	1.138	1.707	2.845	11370	6.26
	10	0.429	1.717	2.576	4.293	9811.75	−8.31
	20	0.517	2.067	3.101	5.168	8664.94	−19.02

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Fig. 5 Percentage changes of parameters vs change of total cost

same is true for the production rate of the rework item (P_1), i.e., if P_1 increases the total cost of the inventory also increases. This is true because manufacturer will produce more reworked item in short time then the machinery costing increases. While if the production rate of the manufacturer decreases then the total cost increases. This is true since the demand of the retailer is price- and stock-dependent so if the manufacturer take more time to produce the finished item then there is a gap in supply from manufacturer to retailer or a chance of shortage which result to increase in the total cost.

(ii) We observe from the table that as the demand parameters (a and b) increases the total cost increase and vice versa. This holds in reality because if we have more stock and if the price of the item are increased then the demand decreases thus the total costing increases.

(iii) As θ increases the total cost decreases moderately. As θ increases then g(z) decreases means rate of defective item decreases which in turn decreases the total inventory costing.

(iv) As λ increases the total cost decreases moderately. As λ increases then f(z) decreases means demand decreases which in turn decreases the total inventory costing.

Case 2. We optimizes the total cost by considering the production rate P_s as the decision variable (Fig. 6).

Here we have considered $t_s = 2$, then the optimized total cost $TC^* = 7165.85 \ and \ P_s^* = 0.11$. We obtain the graph as:

Supply-Demand Relation

Since our model is a three layer supply chain system so a supply demand relation should be maintained, i.e, item should be produced in such a quantity that it should satisfy the demand and there is no shortages. A supply chain is made up by multiple actors, multiple flows of items, information and finances. Each network node has its own customers' and suppliers' management strategies, demand arrival process and demand forecast methods, inventory control policies and items mixture. In real life scenarios,

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Fig. 6 Production rate of supplier (P_s) vs total inventory cost (TC)
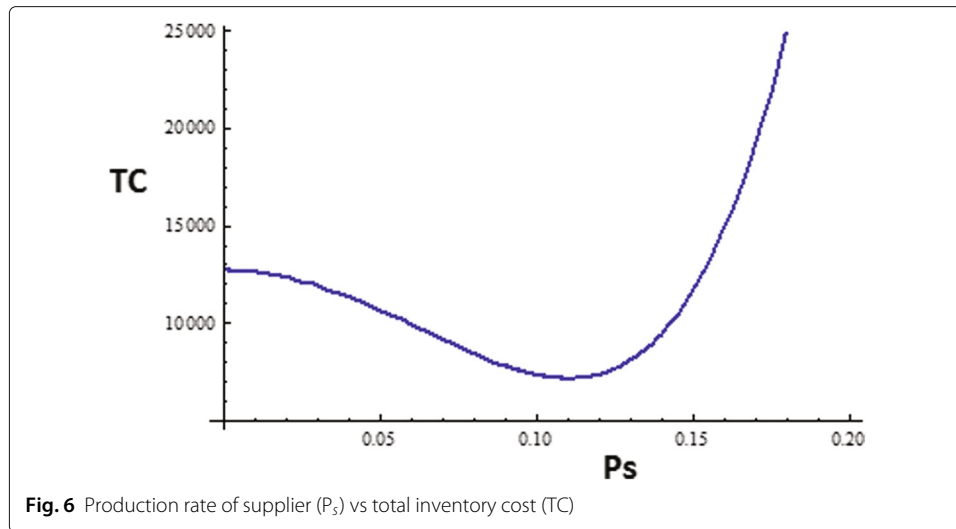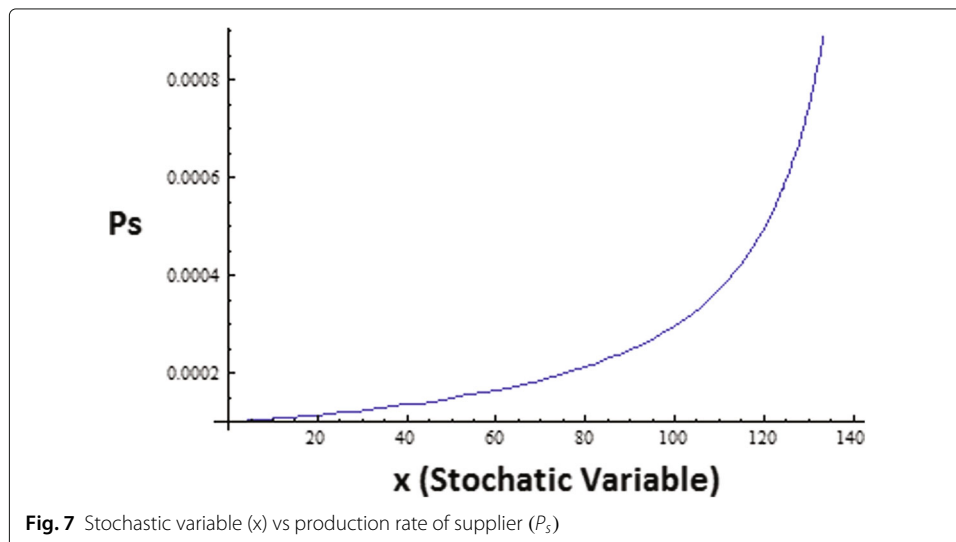
it is often observed there are imbalances in supply and demand of items which results in loss of business and profit simultaneously. It is very difficult for the manufacturer to predict the stochastic demand and plan its raw material accordingly to meet the orders deadline. Again suppliers has its own limitation of production capacity with which it can deliver its order in required time limit. So, here we have tried to draw a relationship between the stochastic demand of the retailer and the production rate of supplier so that we can optimize the inventory in such a way keeping in mind the limitations of supplier's end (observe Fig. 7).

Thus quantity produced by the manufacturer $= \sum\limits_{i=1}^{n} \int_{iT_R}^{(i+1)T_R} D(t,x)dt$

The number of items produced from the raw material of the supplier $= kP_s t_s$

Thus , $kP_s t_s = \sum\limits_{i=1}^{n} \int_{iT_R}^{(i+1)T_R} D(t,x)dt$

Solving the above equation we get $P_s = \frac{2kr^2 P_m^2}{(x-bs)(a-1)n^2 t_s}$

Fig. 7 Stochastic variable (x) vs production rate of supplier (P_s)

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Conclusions

Supply chain optimization is a modern subject. Here, we have not just focused on the retailer's cost, but examined costs in three layers of supply chain. In this paper, we have optimized the whole supply chain, and make it more effective and efficient. Production quality of a supply chain directly effect the coordination of the product flow within a supply chain. Thus, dealing with imperfect items have become an important and growing area of research. The model can be used in textile industries, footwear, chemical, food, cosmetics, etc. where the defective items will be produced in each cycle of production. In our paper, we have assumed that the produced lot at each stage is sent to the subsequent stages in two ways: firstly from supplier to manufacturer where shipments are sent as soon as they are produced and there is no need to wait until a whole lot is produced and secondly from manufacturer to retailer in variable shipment (depend upon customer's demand). This policy leads to considerable savings as compared to the scheme that allows shipments only after the whole lot is produced. In the numerical example, we have formulated and accounted for all the cost in each layer and tried to optimize the total cost with two decision variables t_s (production time of the supplier) and P_s (production rate of the supplier) in two separate cases. From the sensitivity analysis in case 1, we have found that the total cost of supplier is less sensitive, manufacturer is moderately sensitive and retailer is highly sensitive to various parameters. It is also observed from the analysis that not only production rate of supplier and manufacturer can be a decisive factor in optimizing the total cost of inventory, but also the purchase cost, holding cost and sale price of item for retailer can be a major breakthrough in expanding the profit of business in real terms. The major aim of the work presented in this paper is to provide a cost-effective approach that would enable organizations to compete in the global market by coordinating the supplier, manufacturer and retailer when the demand is uncertain. Also reworking the imperfect item is essential. In our paper we observe that it would be better to rework at slower rate which indirectly means less amount of defective item are produced by manufacturer. It is also observed that if the demand increases the supplier has to produce more item but upto certain rate so that the total cost is minimum. Since from case 2 we observe that if the production rate of supplier is increased, the total cost first decreases then it increases. The researchers can work more by considering the effect of shortages and backordered or can consider the effect of inspection error.

Authors' Contributions

GM proposed the framework of the study. SP drafted the manuscript, conceived the study, and participated in its design and coordination. GS participated in the critical analysis of the study. All authors read and approved the final manuscript.

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