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Predicting Human Interest: An Application of Artificial Intelligence and Uncertainty Quantification

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Abstract

The idea that a machine can numerically estimate the interest of an individual towards any entity (e.g., WhatsApp, Facebook) is fascinating. Interest, however, is a complex human property that cannot be quantified by another person; to have a machine-driven method quantify this unobservable and intangible internal property is challenging. In this paper, we make an attempt to address this issue. We propose a novel approach to estimate this internal state of a human. We formulate the interest prediction problem as a hidden state estimation problem and deduce a solution through Bayesian inference. In doing so, we apply indirect inference rules to estimate interest from activity. Activity as a consequence of interest is computed via a subjective-objective weighted approach. We further propose a model for interest by taking inspiration from physics. We use mean reverting stochastic procedures to capture the long-term dynamics of interest. With this perspective, a solution is provided via Monte Carlo simulations. To demonstrate the feasibility of the framework, we develop a web-based prototype and experiment with real-world datasets.

Keywords: Interest prediction, Data engineering, Interest, Uncertainty quantification, Artificial intelligence

Introduction

To numerically estimate interest towards any entity is one of the challenging problems in literature. Through one's social experience, one has often come across the following question: How "much" are you interested in Facebook, WhatsApp, Twitter (or anything)? In other words, we are asking to quantify interest towards any object in the real world. Simply put, we are asking to find a number for someone's interest. Indeed, the question is simple and straightforward. However, we know that the answer to this trivially formed question is challenging. To think of this purely as a human being, we ourself cannot precisely answer the question, it is undeniably a challenge to have a *machine* quantify interest.

Interest is an old topic of research. According to [1], interest is an everyday term used to describe the preference of a person towards real-world objects. It has also been specified that interest is an active propulsive state that is aligned towards real entity, subject, topic, activity, etc. and has a high personal definition [1–4]. We must specify here that there are ample definitions of interest in literature; moreover, the notion spans a broad spectrum with work refining and contextualizing the idea over time, for instance, some authors call it as a mental state [5], an affective state [6], others even mix it with intrinsic motivation [1]. However, in this paper, we work along the definition proposed in [7–10] and call interest as a persistent cognitive and emotional state of mind that makes one to engage, get inspired, sometimes even compels one to take actions towards the object of his/her interest. Interest motivates a person to extend his/her normative capability, thereby reaching the extent of exception, hence, indicating the presence of some very promising feature of the human spirit.

After introducing the notion of interest, let us revisit the question asked in the first paragraph of this paper. That is, if a person is interested in any worldly object, then can we have a method that can model and correspondingly quantify a person's interest. It should be noted here that we are neither asking for a method that can predict one's *level of interest* (e.g., high, low, medium) nor do we want to deduce one's *topic of interest* (e.g., what type of games does one like). Literature has thoroughly investigated these two topics in the context of machine learning, e.g., to infer one's level of interest, there is a plethora of work in machine learning [6, 11–13]. Similarly, for deducing one's topic of interest, there is a dedicated body of work [14–17]. But, in contrast to these works, we are asking for a method that can statistically model, thus, give us a number for interest. Moreover, and in contrast to this *application-specific* line of work, we are interested in quantifying interest towards *any entity*, thereby making the procedure of interest quantification application independent. In non-technical terms, we have raised the following concern: How "much" is one interested in any entity? If we take a close look at the problem, we see that there are several challenges. These are as follows:

1) Interest in its innate form has a tendency to evolve. We have experienced numerous times that interest (in any object) has an inherent characteristic, even a natural propensity, to evolve itself. To exemplify this, a person was highly interested to participate at Facebook, for example, but the desire to engage in the daily activities decreased over time (for any reasons). Therefore, the issue here is *How to model the evolving dynamics of interest* especially considering the typical erratic and unpredictable circumstances in one's routine.
2) We know, and it has been verified in literature, that interest in an object makes a person put in extra efforts and take actions. Naturally, these actions are visible in the form of activity [18–21]. However, an appropriate question in this context is *How to measure activity.* If we take a close look at the term (activity), we realize that it is an abstract concept. A person always expresses his/her actions through multiple viewpoints. That is, activity spans several perspectives that clearly specify that the actions, stimulated by interest, are not limited to a single dimension. Therefore, how to transform the idea of activity, an abstract term having a wide array of dimensions, into a computationally operable construct?
3) *How to model the transformation of interest into activity?* It was discussed in the previous point that interest results into actions. However, the transformation dynamics of interest into activity is unknown. In other words, we do not have a statistical procedure that can model the transformation of interest into activity.

The set of problems identified in the above three points are not only theoretically important but also have a significant impact from a practical point of view. In this paper, therefore, we aim to address these issues and make an attempt to take one more step towards estimating the property of interest. We lay the foundation of this paper in finding a solution to the interest quantification problem and try to show a possible roadmap that can facilitate machines to estimate interest automatically. In this regard, the following points summarize the contribution of this paper:

- The problem to estimate interest is formulated as a hidden state estimation problem and a solution deduced via Bayesian inference. We use principles of uncertainty quantification and machine-oriented procedures to infer the numerical value of interest indirectly from activity.
- To provide a computationally feasible method to calculate activity, we use a subjective-objective weighted approach. We combine several different perspectives of activity into a discreet and computationally acceptable construct.
- We draw inspiration from physics and propose a square-root-based mean reverting stochastic procedure to model the dynamics of interest.
- We use a regression model to dynamically transform interest into activity.
- We combine the contribution of the previous points and present a solution via Monte Carlo simulations. We use particle filter to provide a computationally viable solution to the interest prediction problem.
- To demonstrate the viability of the model in real scenarios, we perform experiments on real datasets. With numerical simulations performed on the Stack Overflow databases, we show that model-based procedures are a good way to estimate variables that are not directly observable.
- To validate the proposed framework in practice, we develop a prototype. We implement the framework as a web service and deploy it on a web server. The prototype is developed using RESTful architecture, thereby providing a uniform interface to access the method by any remote or local application.

We must point out that the idea of interest is broad and that the concept has manifold interpretation (theoretical) in literature. Therefore, to study and correspondingly analyze the property of interest via machines, as well as the notion, can be imprecise. However, it captures the phenomenon of practical importance. Interest in any object makes a person engage and put in extra efforts, consequently, there is activity [18–21]. In this context, and drawing on existing terms in artificial intelligence, e.g., [6, 11, 15, 17], the motive of the paper is propose the use of automatic methods to quantify the phenomenon that provokes activity. In this context, the authors of [22] have specified "As a branch of the science of "Big Data", the field of human-interest dynamics is at its infancy". We base the motivation of this paper along these lines and try to complement work in literature by offering it a possible roadmap to study and analyze interest and other related properties through machine-based procedures.

The rest of the paper is organized as follows: In the "Introduction" section, we discuss the main body of work and present the proposed model. Subsequently, we present the results in the "Results" section. As we are trying to predict the internal state of a human, therefore, we discuss the limitations of the framework in the "Discussion and Limitations" section. Finally, we conclude with the future work in the "Conclusion and Future Work" section.

Methods

Interest Prediction: The Bayesian Perspective

In this section, we describe the theoretical foundation of the proposed work. The method takes its inspiration from the Bayesian inference. Bayesian statistics is a branch of study that has found its way in many disciplines and is especially well appreciated in cases of linearity and non-linearity. With respect to the Bayesian inference and considering the case of the interest prediction problem, the goal of the paper is as follows:

Given a series of activity for a person χ measured for m time units, where the activity vector is defined as $A_\chi = \{A_\chi^1, A_\chi^2, \ldots, A_\chi^m\}$, A_χ^m is the numerical value of activity at the mth unit of time, we have to infer the interest vector I_χ, where I_χ is defined as follows:

$$I_\chi = \left\{ I_\chi^1, I_\chi^2, \ldots, I_\chi^m \right\}.$$

Here, I_χ^m is the interest at the mth unit of time.

To begin with the Bayesian inference, the procedure employed in the paper is graphically explained in Fig. 1. As per this figure, and for interest quantification problem, we need two functions, viz., (1) the transformation function and (2) the measurement function. With respect to the first function, we know that interest evolves itself with time. Therefore, to mathematically represent the phenomenon, we need a *transformation function*. This is expressed as follows:

$$I_t = \hat{T}_t(I_{t-1}, \tau_t), \tag{1}$$

where $\hat{T}_t$: $R^q \times R^w \rightarrow R^e$ is called as the transformation function. The function evolves numerical interest values in the interest space, τ_t is i.i.d. noise element with $\{\tau_t \in \mathcal{N}(.)\}$. For the interest quantification problem, the objective is also to transform the interest space into the observation space. In simple words, interest stimulated the self to take actions, thus, there is activity. To computationally map this event, we need a *measurement function*. This is defined as follows:

$$A_t = \hat{m}_k(I_t, \hbar_{ot}) \tag{2}$$

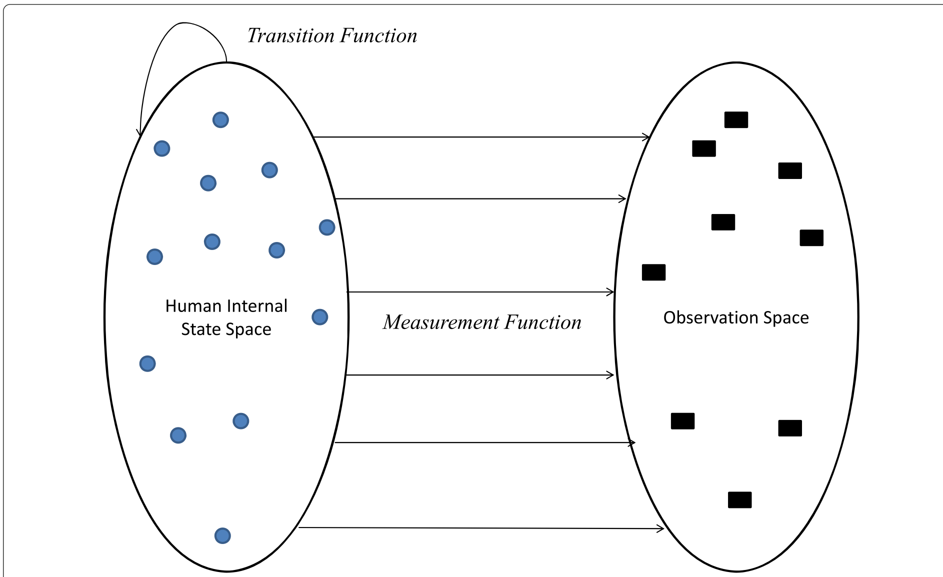

Fig. 1 Bayesian inference for interest prediction problem

where $\hat{m}_t$: $R^q \times R^\nu \rightarrow R^p$, $\hbar_{ok}$ is i.i.d error.

Further, it is common in practical Bayesian inference problems that the two specified functions are parameterized. In this regard, let us assume that the functions are parameterized by γ, also called as the parameter vector. Following the rules of Bayesian statistics, we first have to obtain the prior. This is done using Chapman-Kolmogorov equations:

$$p\left(I_t|A_{t-1},\gamma\right) = \int_I p\left(I_t|I_{t-1},A_{t-1},\gamma\right) p\left(I_{t-1}|A_{t-1},\gamma\right) dI_{t-1} \tag{3}$$

Bayesian inference works in two stages: (1) predict and (2) update. Using Eq. (3), we predicted the interest's value from activity. In the update stage, as soon as new observation (regarding activity) is fed to the algorithm, we modify the current information and calculate the posterior probability density. We use the following equation for this:

$$p\left(I_t|A_t,\gamma\right) = \frac{p\left(A_t|I_t,\gamma\right) p\left(I_t|A_{t-1},\gamma\right)}{p\left(A_t|A_{t-1},\gamma\right)} \tag{4}$$

The denominator is expressed as

$$p\left(A_t|A_{t-1}\right) = \int_I p\left(A_t|I_t,A_{1:t-1},\gamma\right) p\left(I_t|A_{1:t-1},\gamma\right) dI_t \tag{5}$$

From Eqs. (3) to (5), we now have a theoretical understanding of the problem and its possible solution. The next step is to provide a computationally feasible method. To do that, we need specific definitions for the measurement function, the transformation function, a procedure to calculate activity, and a Bayesian filter. We define each of these components in the following sections. We start with the procedure to calculate activity.

Computationally Measuring Activity

It was discussed in the "Introduction" section that interest transforms into activity. However, we also pointed out that activity is an abstract concept. In this section, we present a method to computationally measure activity. In this context, let us understand the method from a general point of view. If we go into the details of the term activity, we realize that it is a concept that has several aspects. To exemplify this, consider the case where one is interested in a mobile game. In this case, the possible perspectives of activity are as follows: the amount of time spent playing, the number of gaming sessions in a day, the gap between subsequent sessions, and so on. From this use case, we can see that interest facilitates actions that span a broad array of perspectives. In other words, there are several points of view of showing interest; therefore, we must record activity by taking into account these different perspectives. In this regard, let us consider that activity has z different perspectives. Mathematically, activity at any time period Φ is a function of z attributes. This is expressed as

$$A_\chi^\Phi = \hat{l}\left(a_1^\Phi, a_2^\Phi, \ldots . a_z^\Phi\right) \tag{6}$$

where the attribute a_b^Φ denotes the bth perspective of activity at time Φ, e.g., if we consider the previous use case (a person is interested in a mobile game), a_b can denote the amount of time spent playing (on any day), a_{b+1} can denote the number of gaming sessions, and so on.

After discussing the perspectives of activity, the next objective is to define the function $\hat{l}$. To that end, we choose a weighted approach [23, 24]. Using this method, we get the following equation:

$$\hat{l}\,(a_1, a_2, \ldots . a_z) = \sum_{i=1}^{z} w_i^A a_i \tag{7}$$

where $w_i^A \in \{0, 1\}$ is the weight of the ith attribute a_i and $\sum_{i=1}^{z} w_i^A = 1$.

It should be noted here that for the function $\hat{l}$, the attributes (or perspectives) of activity are always context and application dependent. Moreover, they must be considered separately for every object of interest. For instance, consider the scenario where a person is interested in a social networking website (for any reason), the possible perspectives of activity are the following: the number of messages, the number of profiles browsed, the number of times a user logged in, the duration of each login session, etc., whereas when a person is interested in an outdoor sport (e.g., football), the perspectives could be the amount of time spent playing, time spent practicing, time spent learning game strategies, and so on. To generalize this behavior, we can see that we do not have a single, definite, and universal attribute set for every aspect and application-dependent perspectives of activity. We have to identify the attributes and have to measure activity for every application or entity of interest separately.

After outlining the idea of perspectives, let us proceed to the next issue: how to calculate weights (w_i^A). This is because weights specify the numerical preference of a person towards the available perspectives of activity. In practice, there are a number of factors that influence the choice and alignment of a person towards a particular perspective of activity. We have observed it many times that two people interested in the same object need not show the same preference towards a specific perspective. To exemplify this, consider the case of social networking websites, e.g., Facebook, some people can show bias towards the number of messages. That is, if they engage in chats, it signifies a much higher level of interest. On the other hand, some people could show bias towards the number of profiles browsed, i.e., they like to browse the profiles of other people. Similarly, others spend a long duration of time surfing through their respective walls. Another way of stating and generalizing the idea is human behavior is sensitive to a variety of factors that shape one character. Moreover, one's choices are mostly, though not always, influenced by one's subjectivity. Therefore, the method to calculate weights must incorporate the subjective nature of humans. Literature, however, has pointed out that sometimes choices made under subjectivity are not the best [25]. One must therefore also consider the element of objectivity. With respect to this reasoning, we employ a subjective-objective approach for allocating weights. We use the formulations presented in [26, 27] for the purpose.

From the discussion in this section, we have integrated subjectivity and objectivity in the model. Further, we have the different perspectives of activity. Consequently, the numerical value of activity (A_χ^Φ) at any time period Φ is computed as

$$A_\chi^\Phi = \alpha W^S \mathrm{AT}^\Phi + (1 - \alpha) W^O \mathrm{AT}^\Phi \tag{8}$$

where W^S is the subjective weight matrix, W^O is the objective weight matrix, AT^Φ denotes the attribute matrix, and $\alpha \in (0, 1)$ is the bias parameter.

Modeling Human Interest

In this section, we present a method to model interest. More specifically, we define the transformation function shown in Eq. (1). However, before we begin the discussion, we must specify that owing to the limitations imposed by the current literature, we do not have a comprehensive understanding about the continuous and long-term evolution of interest. As a result, we model interest by discussing a few observations, then we present the assumptions of the paper. We begin this section with the observations.

1. *Interest is stochastic.* The motive here is backed by work in analytical psychology where internal human processes are often represented as stochastic procedures, e.g., recognition [28]. We can therefore expect interest to be stochastic. If, however, this is false, then interest is deterministic, and we can predict human behavior at any time. One can see that this is a contradiction. Hence, interest is stochastic.
2. *Interest does not increase continuously with time.* To prove this, let us consider the opposite: interest is an ever-increasing continuous function. However, owing to typical erratic and inevitable circumstances in one's daily routine, the cycle follows an uncertain behavior (for instance, ups and downs). Therefore, using proof by contradiction, interest is not an ever-increasing function.
3. There is *no term called as negative interest.* Mathematically speaking, interest can be zero, i.e., not interested at all, or positive, a factor that specifies some degree of interest. The everyday term negative interest imply one dislikes an entity. This statement clearly indicates the absence of interest.

After outlining the observations, let us proceed to the assumptions.

1. *It is assumed that interest fluctuates around a constant numerical value in the long run.* We have observed and experienced many times that when one engages with an entity (e.g., a video game), then interest is usually high in the beginning, but it stabilizes in the long run.
2. *Interest is assumed to be a diffusion process* (A Markov process without jumps).

With this understanding, let us start the procedure to model interest. In this regard, we draw inspiration from physics. Literature in physics often model some of the most uncertain phenomena using stochastic differential equations (SDEs). An example of SDE is given below:

$$dY_t = \mu Y_t dt + \sigma Y_t dB_t \tag{9}$$

where Y is the stochastic variable, μ denotes the mean, σ corresponds to the volatility, and dB is called as the Weiner process. It is described as

$$dB = \sqrt{dt}\mathcal{N}(0,1) \tag{10}$$

In this paper, we use the concept of stochastic differential equations to model interest. In addition to physics, the motive to look into this procedure is also backed by literature in uncertainty quantification. Work in this discipline often uses similar equations to quantify the so-called unquantifiable properties in the real world, e.g., [29, 30]. One can understand that interest quantification is similar to quantifying uncertainty in the internal human states. Therefore, following this reasoning, we use this procedure to model interest. Specifically, we employ a square root-based mean-reverting stochastic differential equation [31] to capture the dynamics of interest. It is an extension of the famous Ornstein-Uhlenbeck process [32]. The equation for square root-based mean-reverting stochastic procedure is described below

$$dI_t = \lambda(\mu - I(t))dt + \sigma\sqrt{I_t}dB_t \tag{11}$$

In this equation, λ corresponds to the convergence speed, μ is the mean value, σ denotes the volatility, and I_t is the interest value at time t. Further, $\lambda, \sigma, \mu > 0$. The following points summarize the equation in its physical form.

- The equation describes the motion of a particle in space. The movement of the particle follows a random behavior at each interval of time.
- λ denotes the speed of the particle.
- μ is the long-term mean value. μ corresponds to the point in space where the particle will settle down in the long run (this property is called as *mean reversion*).
- σ is the volatility component that controls the extent of randomness in the particle's motion.
- The term $\sigma\sqrt{I_t}$ avoids the possibility of having negative interest values.

The expectation and variance of the equation are as follows:

$$E[I_t] = e^{-\lambda t}I_0 + \mu\left(1 - e^{-\lambda t}\right) \tag{12}$$

and

$$\text{Var}\,[I_t] = E\left[(I_t - E[I_t])^2\right] = I_0\frac{\sigma^2}{\lambda}\left(e^{-\lambda t} - e^{-2\lambda t}\right) + \frac{\sigma^2}{2\lambda}\left(1 - e^{-\lambda t}\right)^2 \tag{13}$$

The transition density for Eq. (11) is

$$I_t|I_{t-1} = ce^{-u-v}\left(\frac{v}{u}\right)^{q/2}B_q\left(2(uv)^{1/2}\right) \tag{14}$$

where $c = 2\lambda/(\sigma^2(1 - e^{-\lambda dt}))$, $u = cI_{t-1}e^{-\lambda dt}$, $v = cI_t$, $q = 2\lambda\mu/\sigma^2 - 1$, and B_q is the modified Bessels function of the first kind of order q. Before we proceed further, let us go back and focus on Eq. (11). An important point of note for this equation is when $t \to \infty$, the process stabilizes around a numerical value (recall that interest stabilizes to a particular value in the long run). From Eqs. (12) and (13), we have

$$\lim_{t\to\infty} E[I_t] = \mu \lim_{t\to\infty} Var[I_t] = \frac{\sigma^2}{2\lambda} \tag{15}$$

Following these mathematical foundations and properties, we use Eq. (11) to capture the dynamics of interest. The formulation of the process, however, is not complete. We must point it out that we are trying to make the procedure of interest quantification automatic; consequently, we need to go into more detail. In this context, and from Eq. (11), we see that the framework is dependent upon three crucial parameters: λ, μ, and σ. We therefore need a method to estimate their values. This procedure is explained in the following subsection.

Parameter Estimation

In literature, there is huge body of work dedicated to the study of parameter estimation for stochastic differential equations (SDEs). It has been argued several times that if the parameters of the equation are correct, we can get good numerical estimates of the modeled phenomenon [33]. Parameter estimation for SDEs, however, is non-trivial (the results are extensively elaborated upon in literature, e.g., [33]). Nevertheless, we must find close-enough values. In this paper, we use one of the approximation techniques. More specifically, we estimate the parameters of Eq. (11) using the method of least squares [34]. The procedure is elaborated upon in the following.

To estimate the parameters, we apply Euler-Maruyama method [35] to Eq. (11) and get the following expression:

$$I_{t+1} - I_t = \lambda(\mu - I_t)\triangle t + \sigma\sqrt{I_t}\epsilon_t \tag{16}$$

where $\epsilon_t \sim \mathcal{N}(0, \triangle t)$. Simplifying the expression, we get

$$I_{t+1} - I_t = \lambda\mu\triangle t - \lambda I_t\triangle t + \sigma\sqrt{I_t}\epsilon_t \tag{17}$$

$$\frac{I_{t+1} - I_t}{\sqrt{I_t}} = \frac{\lambda\mu\triangle t}{\sqrt{I_t}} - \lambda\sqrt{I_t}\triangle t + \sigma\epsilon_t \tag{18}$$

For the parameter estimation problem, we first have to get the estimates $(\hat{\lambda}, \hat{\mu})$ for (λ, μ). For this purpose, we minimize the following objective function:

$$(\hat{\lambda}, \hat{\mu}) = \arg\min \sum_{t=1}^{n} \left(\frac{I_{t+1} - I_t}{\sqrt{I_t}} - \frac{\lambda\mu\triangle t}{\sqrt{I_t}} + \lambda\sqrt{I_t}\triangle t \right)^2$$

Taking the partial derivative of this expression w.r.t λ and μ, setting the derivatives equal to zero and simplifying the result produces the following equations:

$$\hat{\lambda} = \frac{Q}{(n^2 - 2n + 1 - \sum_{t=1}^{n-1} I_t \sum_{t=1}^{n-1} \frac{1}{I_t})\triangle t} \tag{19}$$

and

$$\hat{\mu} = \frac{(n-1)\sum_{t=1}^{n-1} I_{t+1} - \sum_{t=1}^{n-1} \frac{I_{t+1}}{I_t} \sum_{t=1}^{n-1} I_t}{Q} \tag{20}$$

where

$$Q = n^2 - 2n + 1 + \sum_{t=1}^{n-1} I_{t+1} \sum_{t=1}^{n-1} \frac{1}{I_t} - \sum_{t=1}^{n-1} I_t \sum_{t=1}^{n-1} \frac{1}{I_t} - (n-1)\sum_{t=1}^{n-1} \frac{I_{t+1}}{I_t} \tag{21}$$

The estimate $\hat{\sigma}$ for σ is calculated as the standard deviation of the residuals. The final expression is shown below:

$$\hat{\sigma} = \sqrt{\frac{1}{n-2} \sum_{t=1}^{n-1} \left(\frac{I_{t+1} - I_t}{\sqrt{I_t}} - \frac{\hat{\mu}}{\sqrt{I_t}} + \hat{\lambda}\sqrt{I_t} \right)^2} \tag{22}$$

Using this procedure, we have the estimates $(\hat{\lambda}, \hat{\mu}, \hat{\sigma})$ of (λ, μ, σ), hence, we have a data-driven statistical method that can effectively model interest.

Interest Resulting Into Activity

From the discussion in the "Introduction" section, we know that interest results into activity. In this section, we describe the mathematical tool for the objective. Specifically, we define the measurement function of Eq. (2). To formulate the function, we neither assume that interest completely results into activity nor that the measurement function is accurate. Consequently, following the method used in Bayesian inference problems [36], we use a regression model to transform interest into activity. This is expressed as

$$A_j(t) = \Theta_0 + \Theta_1 I_j(t) + \psi_t \tag{23}$$

where $A_j(t)$ is the activity at time t, ψ_t is i.i.d error term, and Θ_0, $\Theta_1 > 0$. Note that the numerical value of Θ_0 & Θ_1 is time dependent. Further, the values are estimated as soon as new information is fed to the algorithm. In other words, we perform *online estimation*. This, however, is comparatively simple. We use least square error [34] for this purpose.

Similar to the parameter estimation problem discussed in the "Parameter Estimation" section, the parameters of Eq. (23) are estimated by minimizing the following objective function:

$$\arg\min \Psi\left(\Theta_0, \Theta_1\right) = \sum_{i=1}^{n} \left[A_i - \left(\Theta_0 + \Theta_1 I_i(t)\right)\right]^2$$

Differentiating the above equation w.r.t. Θ_0 and Θ_1, equating the result to zero, and simplifying, we get the final expressions as follows:

$$\Theta_1 = \frac{\sum_{i=1}^{n}\left(I_i - \bar{I}\right)\left(A_i - \bar{A}\right)}{\sum_{i=1}^{n}\left(I_i - \bar{I}\right)^2} \tag{24}$$

$$\Theta_0 = \bar{A} - \Theta_1 \bar{I} \tag{25}$$

where $\bar{I} = \frac{\sum I_i}{n}, \bar{A} = \frac{\sum A_i}{n}$.

From the procedure discussed in this section, we have the definition for the measurement function. Hence, we have a model-based structure that can estimate interest. However, before we can solve the interest quantification problem, we need one more component. In this context, we direct the attention to the "Interest Prediction: The Bayesian Perspective" section and Bayesian inference. The Bayesian inference problems rely upon three ingredients: (1) the measurement function, (2) the transformation function, and (3) A Bayesian filter. Until now, we have the definition of the transformation function (Eq. (11)) and the measurement function (Eq. (23)). We need a Bayesian filter. For this purpose, we use Monte Carlo simulations. Specifically, we employ particle filters.

Particle Filters

Algorithm 1 Particle Filter.

Input: The Activity Vector, A_ξ, $\xi \in \{1, 2, \ldots, n\}$.

Output: The Interest Vector, I_ξ, $\xi \in \{1, 2, \ldots, n\}$.

1. At $t = 0$, generate a series of Z particles Pr, where $Pr = \{p^1, p^2, .., p^Z\}$, $p^k = (\eta^k, w^k)$. Further, sample each particle as: $\eta_0^k \sim \frac{1}{\sigma_R\sqrt{2\pi}} e^{-(\mu_P)^2/2\sigma_R^2}$.
2. For iterations, $j = 1, 2, \ldots, IT$,
3. For $x = 1, 2, \ldots, Z$, sample $\eta_t^x | \eta_{t-1}^x$ via Eq. (11).
4. Predict activity $\hat{A}_t^x$ for every particle p_t^x via Eq. (23).
5. Set $\hat{\eta}_{0:t}^x = (\eta_{0:t-1}^x, \hat{\eta}_t^x)$ and calculate weight. Weight w_t^x is computed as: $w_t^x = \frac{1}{\sigma_A\sqrt{2\pi}} e^{-\left(A_t - \hat{A}_t^x\right)^2/2\sigma_A^2}$.
6. Calculate the total weight of particles T=$\sum_{x=1}^{Z} w_t^x$.
7. Normalize weights w_t^x=$T^{-1} \times w_t^x$.
8. Resample particles based on importance factor (weights).
9. Measure interest by taking mean of the particles $p(\eta_t) \in Pr$.

Go to next Iteration.

end iteration.

Estimate the parameter of Eqs. (11) and (23).

Particle filters (PFs) are probabilistic algorithms that are frequently encountered in uncertainty quantification and Bayesian inference problems. They are a member of the Monte Carlo class of simulations. PFs have been found to be highly efficient in cases where the underlying structure of the model is not accurate [36]. This property is especially beneficial for the interest quantification problem as *precise* evolutionary dynamics of the model are unknown. PFs target high-density areas of the interest space to compute close numerical values. The algorithm represents the posterior by a group of particles and a set of associated weights. As the state space increases, the particles converge to the approximate posterior density, thereby producing good numerical estimates of interest. For this purpose, the system is provided a set of particles, Pr, at time t, $Pr = (\eta_t^x, w_t^x)$. Here, $x = \{1, 2, ..Z\}$, Z = the number of particles, η_t^i represents a numerical hypothesis for interest, and w_t^i is the weight (or the importance factor) of the xth hypothesis with $\sum_{x=1}^{Z} w_t^x = 1$. The weights are chosen via importance sampling (step 5). PFs propagate each probable estimate sequentially and support every hypothesis using the importance factor, thereby providing good approximate values for interest. For reasons of brevity, the exact procedure of the particle filter with encoded definitions of the transformation function and the measurement function is summarized in Algorithm 1.

Gaps in Activity

Until now, we have discussed a method to infer interest from activity. However, the framework has a shortcoming: The algorithm cannot infer interest in cases of activity gaps. To understand this issue, let us assume a situation where a person is interested in playing football on the field (and has been playing it everyday). But, as is common in practical situations, for several uncertain and unexpected reasons (e.g., one is occupied somewhere), the person is not able to play the game on the "current" day. Thus, there is no activity. However, it is understood that interest is not zero on the day when there is no activity. This use case exemplifies the problem of the activity gap. To tackle this issue, we use the idea of K-step ahead prediction density. In other words, when *continuous* observations are not available, we discard Eq. (3) and use the following theoretical equation:

$$p\left(I_{t+k}|A_{t-1},\gamma\right) = \int_I p\left(I_{t+k}|I_{t-1},A_{t-1},\gamma\right) p\left(I_{t+k-1}|A_{t-1},\gamma\right) dI_{t+k-1} \tag{26}$$

To provide a computationally feasible solution from the above representation, we use Eq. (11) during activity gaps. To put the idea in simple words, when we face the situation of the activity gap, the system automatically evolves interest in the next interval of time. For instance, in the previous use case (where one is interested in playing football), the system automatically uses Eq. (11) on the day the person is not able to engage. In this case, note, we can predict the interest's value, but we cannot update it. However, once new information (about activity) is available, we use Eqs. (11) and (23) to predict and update the interest value using particle filter. Thus, we have a method similar to the continuous time model of interest. This type of modeling is beneficial as we mathematically expect any internal human state to be a continuous time function.

Results

Data Collection and Experimental Setup

To validate the viability of the proposed theory in practice, we experiment with real datasets. More specifically, we use the datasets provided by StackOverflow[1]. This platform is a mature and a highly respected Q&A discussion forum on the Internet. Further, it has one of the largest public data repositories. Owing to these characteristics, it has attracted a good amount of attention in literature. Work has found that the users of StackOverflow are addicted to participate in its daily activities [37, 38]. Therefore, this online platform presents an excellent opportunity to test the feasibility of the model in real scenarios. In this regard, and to test the model, we collected the granular details of 250 users online[2]. This was done on a daily basis for one whole year. As the data was collected on a day-by-day basis, therefore, interest was also estimated daily. For the purpose of discretization (of SDEs), we use the Euler-Maruyama method [35].

In this paper, we have deduced interest from activity. Therefore, the first step in the experimental setup is to calculate activity. Recall that we discussed in the "Computationally Measuring Activity" section that activity depended upon several attributes (or perspectives). In this regard, and for the purpose of experimentation, we collected the following attributes: (1) the number of comments, (2) the number of answers, (3) the number of questions, (4) the number of edits, and (5) the time to answer a question. Owing to reasons of privacy, we could not include more attributes. Nevertheless, with these numerical attributes, the procedure to calculate activity is elaborated upon in the following points.

Activity Calculation

1. We fed the system a pairwise comparison matrix to calculate the value of the weights. This was done by following the method discussed in [26, 27]. The subjective-objective weight matrices obtained after applying the procedure are presented in Table 1.
2. To explain the procedure of activity calculation in detail, an example is discussed in Table 2. In this table, we have presented the case of one random user. Further, and for illustration purposes, we have shown activity calculation for 7 days only (one can generalize the method for any number of days). It was specified in the above paragraph that we collected five different attributes from StackOverflow. They are represented under the column: AT1, . . ., AT5. From this data, we then normalized the attributes between 1 and 10. The normalized attributes are highlighted under the column: NT1, . . ., NT5. The matrix containing these normalized attributes is called as the attribute matrix. Recall that to calculate activity via Eq. (8), we need the attribute matrix, the subjective-objective matrices, and the bias parameter.

Table 1 Subjective-objective weights for the experiment

Attribute	Subjective	Objective
Answers	0.1549	0.5941
Questions	0.1333	0.1166
Comments	0.2127	0.0916
Edits	0.1944	0.0536
Time to answer	0.3047	0.1441

Table 2 Activity calculation

	AT1	AT2	AT3	AT4	AT5	NT1	NT2	NT3	NT4	NT5	Activity
Day 1	1	1	1	1	46	1	1	1	1	1.003	1.0005
Day 2	1	1	1	1	15	1	1	1	1	1	0.999
Day 3	1	1	1	1	236	1	1	1	1	1.0211	1.0043
Day 4	1	1	1	1	156	1	1	1	1	1.0138	1.0027
Day 5	5	1	1	2	91,920.2	10	1	1	10	10	7.63
Day 6	4	2	1	1	32.5	7.75	10	1	1	1.001	4.933
Day 7	3	1	2	1	113	5.5	1	10	1	1.009	4.1451

Subjective weight matrix: [0.1549, 0.133, 0.2127, 0.1944, 0.3047], objective weight matrix: [0.5941, 0.1166, 0.0916, 0.0536, 0.1441], bias parameter $\alpha = 0.6$; activity for day 1 was calculated as: $0.6 \times (0.1549 * 1 + 0.133 * 1 + 0.2127 * 1 + .1944 * 1 + 0.3047 * 1.003) + (1 - 0.6) \times (0.5941 * 1 + 0.1166 * 1 + 0.0916 * 1 + 0.0536 * 1 + 0.1441 * 1.003) = 1.0005$
Abbreviations: *AT1* attribute 1, *AT2* attribute 2, *AT3* attribute 4, *AT5* attribute 5, *NT1* normalized attribute 1 , *NT2* normalized attribute 2, *NT3* normalized attribute 3, *NT4* normalized attribute 4, *NT5* normalized attribute 5

3. From steps 1 and 2, we obtained the attribute matrix and the subjective-objective weight matrices. Further, with the bias parameter (α) as specified in the table, we used Eq. (8) to calculate the numerical values of activity. The resulting activity vector is shown in the table under the heading activity. An example for day 1 is also explained in the table.
4. Steps 1–3 were followed on a daily basis for 250 users, thereby the data consisted of 250 activity vectors. The dataset for numerical activity is made public and can be found at https://drive.google.com/file/d/0B-evB9aVUINtbm1YNlZnOVMtVmc/view?usp=sharing.

Once we obtained the activity vector, the subsequent step was to predict interest. The following points summarize the procedure to obtain the numerical interest values.

Interest Estimation

- After calculating activity, we coded the definitions of the transformation function (Eq. (11)) and the measurement function (Eq. (23)) into the particle filter. In terms of Bayesian statistics, the Bayesian filter was encoded with the *state model* and the *output model*.
- Once the definitions of the two functions were encoded, the subsequent step was to feed the particle filter the input data for activity. The data for activity was obtained by following the procedure described under the “Activity Calculation ” section.
- Lastly, we used Algorithm 1 to predict the numerical interest vector for all the 250 users. In other words, using basic rules of the Monte Carlo simulations, we obtained 250 interest vectors.

As is commonly known in Bayesian inference problems, the method is not only able to estimate interest, but it can also predict activity. Therefore, based on the predicted value of activity and the actual activity available to the system, we evaluate the performance of the model. We have chosen the traditional RMSE & MAE as the error metrics. The procedure to obtain the error values is explained in the following points.

Procedure to Obtain RMSE and MAE

1. From the procedure discussed under the heading interest estimation, we obtained 250 interest vectors as well as 250 “predicted” activity vectors. Further, from the procedure explained under the “Activity Calculation ” section, the system had 250

"actual" activity vectors. Therefore, using the basic rules of error calculation, we computed the RMSE and MAE for every user separately.

2. From the previous step, we obtained 250 RMSE and MAE values (one for each user). We then took the average of all the 250 RMSE and MAE values, thereby we obtained only one numerical value for RMSE and MAE.
3. We repeated steps 1 and 2 50 times. As a result, we obtained 50 numerical RMSE and MAE values.
4. We then took the average of the 50 numerical error values obtained from steps 1–3 and obtained a single number. We present this value in the paper. It represents an overall measure of the predictive capability of the system.

Prototype Development

To demonstrate the feasibility of the method in practical deployment scenarios, we have implemented a prototype. The proposed method has been successfully deployed as a RESTful service on a web server. The application was developed using JAX-RS[3]. The application container to host the services was Apache Tomcat v7.0.41. The mathematical functions were implemented from Apache Common Math library[4]. The prototype was developed using JAVA programming language. The configuration of the machine to conduct experiments was Intel i7 processor, 8 GB RAM, and Ubuntu 12.04 LTS OS. A snapshot of the developed application is shown in Fig. 2.

Model Analysis

To begin with the analysis, we have shown the probability distribution function of interest in Fig. 3 (the data in the graph is normalized). Further, the evolution of interest is also presented in Fig. 4. In the two figures, the result for only one random user is presented (all other users follow similar behavior). It is visible from Fig. 4 that the evolution of interest repeatedly follows several ups and downs. Therefore, from this evidence, one can deduce that interest takes on a wide array of numerical values. If we consider typical everyday uncertainty and certain erratic circumstances in one's daily routine, this type of evolution is expected. For instance, if a person is interested in playing a mobile game, then it is logical to expect that the person will not have the same interest in the game for every hour of the day. Moreover, this pattern is in line with the stochastic (and chaotic) nature of interest as discussed in the "Modeling Human Interest" section.

Fig. 2 A snapshot of the developed prototype

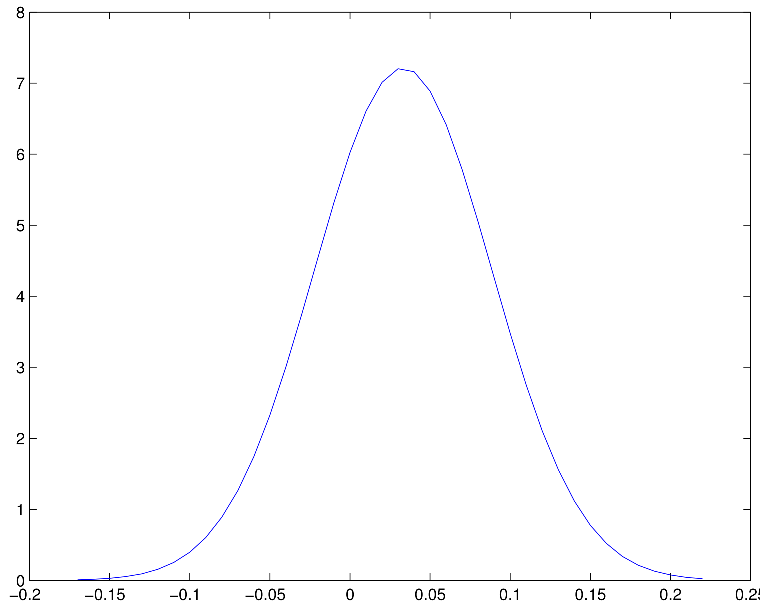

Fig. 3 Probability distribution of interest for one random user

The predictive capability of the framework is shown in Fig. 5. In this figure, the results for activity prediction are presented. We have compared the actual activity with the predicted activity values. For reasons of brevity, the results for only one user is presented. It is visible from the figure that that the result for activity prediction is not accurate. However, from Fig. 5, we can see that the prediction is getting close in the later stages. We must point out that the initial estimates are off margin owing to the problem of parameter estimation. Recall that the framework is dependent upon several parameters. In this regard,

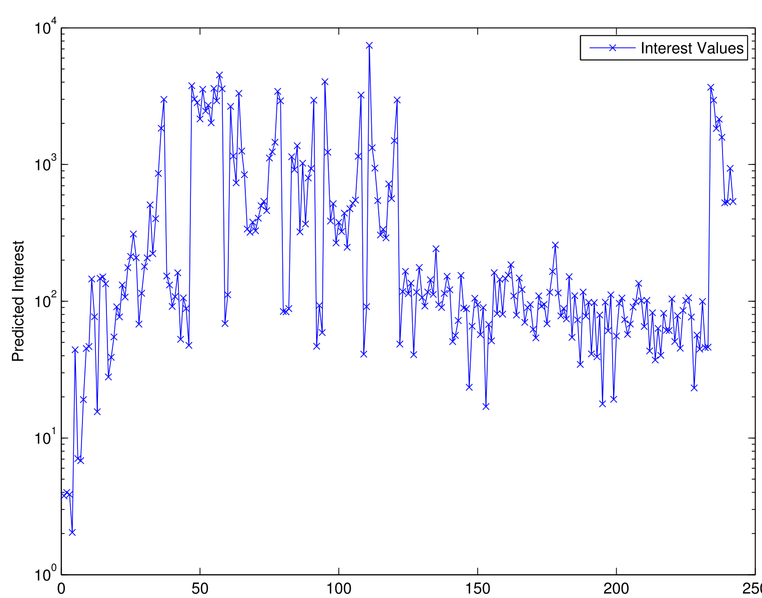

Fig. 4 Evolution of interest for one random user

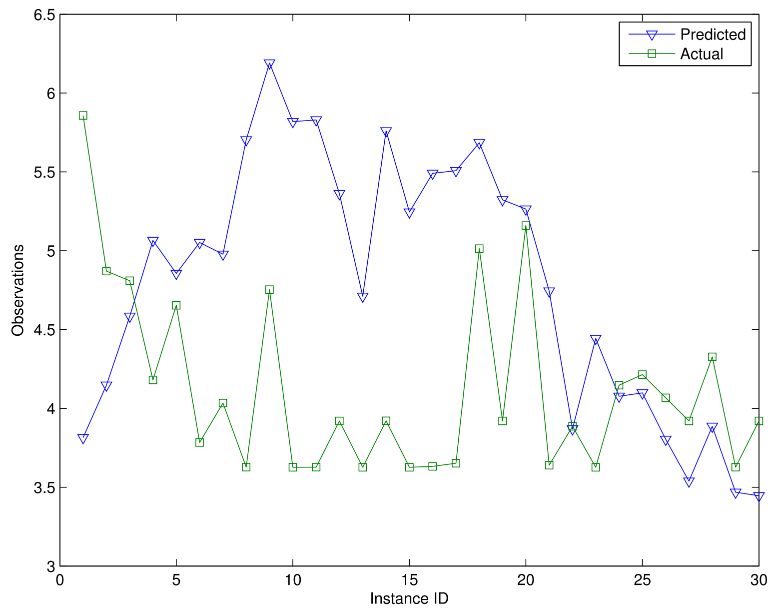

Fig. 5 Predicted observations vs actual observations

it is commonly known that to have a close-enough value of the parameters, we need substantial quantities of data. Moreover, even if we have enough data, parameter estimation (for SDEs) is challenging [39]. In this context, we must also point out that we are dealing with uncertainty quantification in the internal mental states. The objective in such type of problems is to get close numerical approximations [36].

Comparison with Similar Procedures

In this section, we test the performance of the proposed model for interest. More specifically, we test the feasibility of the square root-based mean-reverting stochastic differential equation (Eq. (11)) to capture the dynamics of interest. We have compared the performance of the framework with some of the widely followed procedures in literature. We compare the performance with random walk (RW), geometric Brownian motion (GBM), and the Ornstein-Uhlenbeck (OU) process [32].

To begin with the purpose of comparison, we first predict activity using the proposed model for interest, i.e., Eq. (11). Subsequently, we discard this equation and model the evolution of interest via RW. Similarly, we next model interest via GBM and the OU process. We keep the rest of the procedure as it is (i.e., Eq. (23) and Algorithm 1 is kept intact) and estimate activity. With this experimental setup, the results are shown in Table 3. It is visible from the table that the results for RW and GBM are not satisfactory. These methods are widely applied in many practical applications. Further, they are commonly followed to model the internal mental states (of a human) in several papers, e.g., [40–42], but when we model interest using these methods, the performance is not satisfactory. Specifically, the results for RMSE are compromised. However, when we employ the OU process as a model for interest, we see an improvement. This is because the OU process belongs to the class of *mean-reverting stochastic differential equations*. With respect to this statement, the performance of the proposed method is even better. We get the best performance with the proposed method. If we focus on the numbers, then with respect to RW, GBM,

Table 3 Comparison of the proposed model with similar procedures

	MAE	RMSE
Random walk	1.6702465926	2.763589123
Geometric Brownian motion	1.9440715077	5.025938536
Ornstein-Uhlenbeck process	1.6006146649	1.9056068
Proposed framework	1.46330893	1.78828697

and the OU process, MAE improved by 12.38, 24.72, and 8.57 % and RMSE improved by 35.29, 64.41, and 6.15 %, respectively. These figures demonstrate good performance of the framework.

Impact of Varying the Parameters of Particle filters

In addition to testing the performance of the base model, we also experimented by varying the parameters of the particle filter. In this direction, there are two parameters: (1) the number of iterations and (2) the number of particles. When the number of iterations is high, the particles are given enough time to explore a good solution, whereas if the number of particles is high, then the space exploration of the algorithm is good. However, both the two procedures increase the computation time. In this regard, the result for the former parameter (iterations) is shown in Table 4. In this test, we have varied the number of the iterations from 10 to 100. It is visible from the table that increasing the number of iterations improves the accuracy. However, at the same time, it also increases the computation time. Therefore, the results ask for a compromise between accuracy and computation time.

Similar to varying the number of iterations, we also varied the size of particles. The results for this test is shown in Table 5. From the table, we can see that when we increase the number of particles, we improve the performance of the model. However, in contrast to the results discussed in the previous paragraph, the improvement is by a small margin. But it is visible from the table that the execution time increased by a good number. Thus, from the results presented in Tables 4 and 5, one has to find an optimal balance between particle size, iteration count, and execution time.

Discussion and Limitations

In this section, we analyze the practical implications of having a computational agent deduce interest and, at much the same time, we examine the limitations with the proposed work.

1. Theoretically speaking, the method can quantify interest from measurable activity. However, from a practical point of view, it is not universal. The proposed model

Table 4 Accuracy and execution time for different iterations. Number of particle = 20

Number of iterations	MAE	RMSE	Execution time (ms)
10	1.8205927612	2.0737191934	26,017
20	1.4398928187	1.7549857745	50,180
30	1.1140898629	1.5255709094	70,207
40	0.8602689489	1.3382325218	89,795
50	0.6233805855	1.2421697673	103,734
100	0.2821038912	1.0196928238	178,232

Table 5 Accuracy and execution time for different numbers of particles. Iteration = 10

Number of particles	MAE	RMSE	Execution time (ms)
10	1.8206345202	2.0835584618	12,818
20	1.8198054154	2.0735584618	24,876
30	1.8161826225	2.0460381487	31,761
40	1.8139458681	2.0417554565	38,753
50	1.8132130837	2.0382101465	45,859
100	1.8130539885	2.0361421347	86,864

will fail in situations when the system cannot measure activity. The prevailing technology is not advanced enough to observe activity for every possible application and object of interest, for instance, we cannot measure activity in the case when a person is interested in reading books. In this scenario, the system is incapable of recording activity; therefore, we do not have data. The presence of data is imperative for the proper working of the system. Hence, for this and similar situations, we cannot estimate interest.

2. Connected to the previous problem is the situation where a person is interested in an entity but has not taken any steps to express interest. We understand that interest is an intangible mental variable that we are trying to model via computational approaches. Therefore, we have to face several mechanistic realities. That is, if we expect to estimate interest via machines, then data must be fed to an algorithm. If interest is only considered as an inner feeling, as something that can be expressed without any perceptible medium (or media), then we cannot expect a computational agent to quantify this construct. In this regard, and similar to the previous point, we need data to support the operations of the system. The absence of which leaves the system incapable of working.
3. One can deduce that through the procedure employed in this paper, we get a number for interest. But a good question is *What does the number imply*. This question has two points of view.

    - First, *comparing the interest of users*. It should be noted here that there is no criterion to compare the interest of people on a common scale. For example, a user has an interest of 0.1, whereas another one has an interest of 0.4 (towards a common entity). This does not imply that the second user is more interested in the entity of common interest. We cannot compare the interest of one individual with the interest of another by weighing them both on the same scale. The rationale here is backed by work in psychology. To explain the idea, we quote a few words: Interest is an active propulsive state that is aligned towards real objects, and has a *high personal definition* [4]. In the context of these lines, we must specify that interest has a "personal" meaning. In simple words, every user has his/her own way to express interest. For the interest quantification problem, we must not make the mistake of defining a common criterion to measure interest.
    - The second point of view is *How to devise an algorithm that can understand or feel the number*. This question is rather tough to answer. Although, literature in affective computing have devised several methods for emotion quantification, but an accurate computational method that can

understand/feel *human like emotions*, or any other internal mental state, is tough to implement. Nevertheless, we must point out that we have not tried to answer this question. We have tried to find the number.

4. The last problem is the mathematical framework.

    - We have modeled interest using a square root-based mean-reverting stochastic differential equation. Mean-reverting stochastic differential equations are employed in a plethora of work that deals with uncertainty, e.g., [43–46]. However, it is not claimed here that the proposed model is accurate. Interest quantification is a challenging task. We need more efforts. Moreover, efforts need not be limited to mean-reverting procedures.
    - The second problem is parameter estimation. Accurately predicting parameters of SDEs is a standing problem in literature. Work has specified: if the parameters are precise, we can obtain good approximates of the underlying phenomenon [39]. However, parameter estimation is non-trivial. We therefore employed one of the approximation techniques. Consequently, the method suffers from performance issues. Nevertheless, the numbers presented in the "Results" section show acceptable performance.
    - We modeled the transformation of interest into activity through a regression model. To back this rationale, we quote a few words: "Existing computational assume a positive correlation between stimulation and curiosity" [47]. In this paper, we worked along the idea behind these lines. Although it is acceptable that interest and activity can have a positive correlation, but we do not claim that this idea is universal. One has to understand that similar to modeling interest, engineering a statistical procedure to transform interest into activity is also non-trivial. We need more efforts to statistically understand the way interest transforms into activity.

Conclusion and Future Work

In this paper, we proposed a method to model and correspondingly estimate interest using statistical procedures. Interest prediction problem was formulated as a hidden state estimation problem, and a solution was provided via Bayesian inference. Activity was calculated via a subjective-objective weighted approach. Subsequently, indirect inference rules were employed to infer numerical estimates of interest from activity. A model for interest was proposed by drawing inspiration from physics. Interest was modeled as a square root-based mean-reverting stochastic procedure. Particle filter was employed to provide a computationally feasible solution to the problem. A prototype was developed and experimentation was performed on real datasets. Through numerical investigation, it was found that the procedure showed acceptable performance. Several limitations of the proposed method were discussed in detail.

The work presented in this paper just scratched the surface on estimating and modeling interest. For the future work, we need to improve the accuracy of the model. Purely from an engineering point of view, we saw that mean-reverting stochastic procedures are a good option to model interest. However, we need to test the mathematical aspects of the theory in detail. Moreover, we need to look into advanced procedure to model the evolution of interest.

Endnotes

[1] http://www.stackoverflow.com

[2] http://data.stackexchange.com/

[3] https://jersey.java.net/

[4] http://commons.apache.org/proper/commons-math/

Authors' contributions

TA carried out the study in the paper and drafted the first version of the manuscript. TA and AS designed the framework and developed the proposed method together. AS revised the first version of the manuscript. Both authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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