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Dependent-Chance Programming on Sugeno Measure Space

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Abstract

In order to solve the optimization problem of selecting the decision with maximal chance to meet the Sugeno event in Sugeno environment, dependent-chance programming on Sugeno measure space is proposed, which can be considered as a generalized extension of the stochastic dependent-chance programming. Firstly, the theoretical framework of dependent-chance programming on Sugeno measure space is established. Secondly, a Sugeno simulation-based hybrid approach, which consists of back propagation neural network and genetic algorithm, is presented to construct an approximate solution of the complex dependent-chance programming models on Sugeno measure space. Finally, some numerical examples are given to illustrate the effectiveness of the approach.

Keywords: Sugeno measure space, Dependent-chance programming, Sugeno simulation, Hybrid approach

Introduction

There are a lot of uncertainties in decision sciences, engineering, information sciences, system sciences, etc. By uncertain mathematical programming, we can solve optimization problems under uncertain environment. The first method of uncertain mathematical programming is the expected value model (EVM) [1–4] which optimizes the expected objective functions to satisfy some expected constraints. The second method is named chance-constrained programming (CCP) [5–10] which is a way to solve optimization problems by assigning a confidence level at which the constraint holds. Occasionally, a complex decision system undertakes multiple tasks called events, and the decision maker wishes to maximize the chance functions of satisfying some events [11]. In order to solve the problems, Liu initiated the third method of uncertain mathematical programming named dependent-chance programming (DCP). He firstly proposed the DCP in stochastic environments [12], and then, he gave the theoretical frameworks of DCP in fuzzy environments [11, 13], random fuzzy DCP [14], and fuzzy random DCP [9]. In the past few years, DCP has been used to solve many optimization problems, such as the dynamic facility layout problem [15], the bi-level resource-constrained project scheduling problems [16], and the inventory modeling problems without and with backordering [17].

In spite of multiple DCP being made, there also exist some limitations. For example, stochastic DCP is established on the basis of the probability measure which should satisfy the additivity, and fuzzy DCP deals with the problems containing fuzziness. But in reality, this requirement for additivity cannot be easily satisfied or might not be satisfied at all [18]. In addition, we may deal with problems without fuzziness. Therefore, we introduce DCP on the Sugeno measure space. Sugeno measure space is one measure space based on Sugeno measure. Sugeno measure is one type of representative non-additive measures and an important generalization of probability measure [19]. Let us give an example of purchasing apples to illustrate the point. For convenience, let the universe of discourse consist of two properties characterizing the apples such as suitable price (a) and suitable quality (b), say $X = \{a, b\}$. Let $P(X)$ denotes the power set of X and μ describes an importance degree or a purchasing possibility of various elements of $P(X)$. Apparently, apples with too high price (no suitable price) and too low quality (no suitable quality) will not be purchased. In this case, the purchasing possibility is equal to 0. Moreover, we will purchase apples with suitable price and suitable quality. In this case, the purchasing possibility is equal to 1. Usually, the quality is more important than the price, so this might result in purchasing possibilities of purchasing apples with only suitable price and only suitable quality are 0.5 and 0.2, respectively. Let

$$\mu(E) = \begin{cases} 0, & E = \phi \\ 0.5, & E = \{a\} \\ 0.2, & E = \{b\} \\ 1, & E = X. \end{cases}$$

This measure could express the subjectivity permeating above problem. Evidently, the above measure μ is nonadditive ($\mu(X) \neq \mu(\{a\}) + \mu(\{b\})$), that is, μ is not a probability measure. We can show that μ is a Sugeno measure with $\lambda = 3$ [19]. Sugeno measure and Sugeno measure space have been researched by many scholars. Wang and Klir [19] gave the basic definitions and properties of Sugeno measure. Ha et al. [18] proposed the key theorem and the bounds on the rate of uniform convergence of learning theory on Sugeno measure space. Ha et al. [20, 21] gave the key theorem and the theoretical foundations of statistical learning theory based on fuzzy random samples in Sugeno measure space. Shi and Gao [22] researched on quality evaluation of lexical cohesion based on Sugeno measure. Zhang and Zhang [23] proved Borel-Cantelli lemma for Sugeno measure. In order to solve the optimization problems on Sugeno measure space, Ha et al. [24] proposed the expected value models on Sugeno measure space and Zhang et al. [25] proposed the chance-constrained programming on Sugeno measure space. The elemental concepts and properties were given, and hybrid algorithms to solve the above programming were proposed.

The remainder of this paper is organized as follows. "Preliminaries" section discusses the g_λ variable and its characterization, redefines its expected value and variance, and then revises the strong law of large numbers in [24]. "Dependent-chance Programming on Sugeno Measure Space" section firstly proposes the concepts of Sugeno environment, event and chance function, and then gives the principle of uncertainty which is the theoretical basis of the DCP on Sugeno measure space. At the end of this section, the theoretical framework of the DCP on Sugeno measure space is established. "A Hybrid Approach to Solve the DCP on Sugeno Measure Space" section gives a Sugeno simulation-based hybrid approach which consists of back propagation (BP) neural network and genetic algorithm (GA) to solve DCP on Sugeno measure space. Section 5 provides numerical examples to illustrate the methodology and effectiveness of the approach. Finally, conclusions are drawn in "Numerical examples" section.

Preliminaries

For the sake of convenience and completeness of our investigations, we offer some basic definitions and properties.

Definition 1 [19] Let X be a nonempty set, ζ be a nonempty class of subsets of X, and μ be a nonnegative real valued set function on ζ. If there exists $\lambda \in \left(-\frac{1}{\sup\mu}, \infty\right) \cup \{0\}$ where $\sup \mu = \sup_{E \in \zeta} \mu(E)$ such that

$$\mu\left(\bigcup_{i=1}^{\infty} E_i\right) = \begin{cases} \frac{1}{\lambda}\left\{\prod_{i=1}^{\infty}[1+\lambda \cdot \mu(E_i)]-1\right\}, & \lambda \neq 0 \\ \sum_{i=1}^{\infty} \mu(E_i), & \lambda = 0 \end{cases}$$

for any disjoint class $\{E_n\}$ of set in ζ whose union is also in ζ, then we say that μ satisfies the $\sigma-\lambda$ -rule (on ζ).

Definition 2 Let $\mathscr{F}$ be a σ-algebra of subsets of a nonempty set X and μ be a non-additive real valued set function on $\mathscr{F}$. Then, μ is called a Sugeno measure, if it satisfies the σ-λ-rule and $\mu(X) = 1$ [18]. Usually, Sugeno measure μ is denoted by g_λ. Then, the triple $(X, \mathscr{F}, g_\lambda)$ is called a Sugeno measure space [19].

Obviously, g_λ is a flexible non-additive measure due to the parameter λ which could take different numeric values [18]. When $\lambda = 0$, g_λ reduces to probability measure and a Sugeno measure space reduces to a probability measure space. Therefore, we stipulate that $\lambda \neq 0$ in the remainder of the article.

The following theorem shows the transformations between Sugeno measure and probability measure.

Theorem 1 [19] *If g_λ is a Sugeno measure and*

$$\theta_\lambda(x) = \frac{\ln(1+\lambda x)}{\ln(1+\lambda)}\left(x \in \left(-\frac{1}{\lambda}, +\infty\right)\right),$$

then $\theta_\lambda \circ g_\lambda$ is a probability measure.

Conversely, if P is a probability measure and

$$\theta_\lambda^{-1}(x) = [(1+\lambda)^x - 1]/\lambda(x \in (-\infty, +\infty)),$$

then $\theta_\lambda^{-1} \circ P$ is a Sugeno measure.

Definition 3 [18] Let $(X, \mathscr{F}, g_\lambda)$ be a Sugeno measure space. A function $\xi : X \to R$ is called a g_λ variable if $\{\omega | \xi(\omega) \le x\} \in \mathscr{F}$ for all $x \in \mathfrak{R}$.

Definition 4 [18] The Sugeno distribution function of a g_λ variable ξ is defined as

$$F_{g_\lambda}(x) = g_\lambda\{\xi \le x\}, \forall x \in \mathfrak{R}.$$

Example 1 [24] A g_λ variable ξ has a Sugeno normal distribution if its Sugeno distribution function is

$$F_{g_\lambda}(x)=\begin{cases}\dfrac{1}{\lambda}\left\{(1+\lambda)^{\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{x}e^{-\frac{(t-\mu)^2}{2\sigma^2}}dt}-1\right\}, \lambda\neq 0\\ \dfrac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{x}e^{-\frac{(t-\mu)^2}{2\sigma^2}}dt, \qquad \lambda=0,\end{cases}$$

denoted by $\xi \sim SN(\mu, \sigma^2, \lambda)$, where μ, σ, and λ are all three real numbers.

Example 2 A g_λ variable ξ has a Sugeno λ - 0 - 1 distribution if its Sugeno distribution function is as follows:
when $\lambda \neq 0$,

$$F_{g_\lambda}(x)=\begin{cases}0, & x\leq 0\\ [(1+\lambda)^x-1]/\lambda, & 0<x<1\\ 1, & x\geq 1,\end{cases}$$

and when $\lambda = 0$,

$$F_{g_\lambda}(x)=\begin{cases}0, & x\leq 0\\ x, & 0<x<1\\ 1, & x\geq 1,\end{cases}$$

denoted by $\xi \sim S\mathrm{U}(\lambda)$, where λ is a real number.

We can note that $\xi \sim SN(\mu, \sigma^2, \lambda)$ equals to $\xi \sim N(\mu, \sigma^2)$ and $\xi \sim SU(\lambda)$ equals to $\xi \sim U$ when $\lambda = 0$.

In the following Definitions 5 and 6, the expected value and the variance of a g_λ variable are redefined, which revise the definitions in [24].

Definition 5 Let ξ be a g_λ variable and $F_{g_\lambda}(x)$ be the distribution function of ξ. If $\int_{-\infty}^{\infty}|x|d\theta_\lambda\left[F_{g_\lambda}(x)\right]<\infty$, then we call $\theta_\lambda^{-1}\left\{\int_{-\infty}^{\infty}xd\theta_\lambda\left[F_{g_\lambda}(x)\right]\right\}$ an expected value of ξ and denote it by $E_{g_\lambda}(\xi)$ or $E(\xi)$.

Definition 6 Let ξ be a g_λ variable. If $E_{g_\lambda}\left\{\left[\xi-\theta_\lambda\left(E_{g_\lambda}\xi\right)\right]^2\right\}$ exists, then we call $E_{g_\lambda}\left\{\left[\xi-\theta_\lambda\left(E_{g_\lambda}\xi\right)\right]^2\right\}$ the variance of ξ and denoted it by $D_{g_\lambda}(\xi)$ or $D(\xi)$.

Definition 7 [18] The joint distribution function $F_{g_\lambda}:\mathfrak{R}^2\to[0,1]$ of a g_λ vector (ξ, η) is defined as $F_{g_\lambda}(x,y)=g_\lambda\{\xi\leq x, \eta\leq y\}$, for any $x, y \in \mathfrak{R}$.

Definition 8 [18] The g_λ variables ξ and η are called independent if for all x and y,

$$F_{g_\lambda}(x,y)=\theta_\lambda^{-1}\left\{\theta_\lambda\left[g_\lambda(\xi\leq x,\eta<\infty)\right]\cdot\theta_\lambda\left[g_\lambda(\xi<\infty,\eta\leq y)\right]\right\}.$$

Definition 9 [24] Suppose that $\xi, \xi_1, \xi_2, \cdots$ are g_λ variables defined on the Sugeno measures space $(X, \mathscr{F}, g_\lambda)$. We say that the sequence $\{\xi_n\}$ converges in Sugeno measure to ξ if $\lim_{n\to\infty} g_\lambda\{|\xi_n-\xi|\geq\varepsilon\}=0$ for every $\varepsilon > 0$. In this case, we note $\lim_{n\to\infty}\xi_n=\xi\ (g_\lambda)$ or $\xi_n \xrightarrow{g\lambda} \xi$.

Definition 10 [24] Suppose that $\xi, \xi_1, \xi_2, \cdots$ are g_λ variables defined on the Sugeno measures space $(X, \mathscr{F}, g_\lambda)$. The sequence $\{\xi_n\}$ is said to be convergent almost surely (a.s.)

to ξ if and only if there exists a set $A \in \mathscr{F}$ with $g_\lambda(A) = 0$ such that $\lim_{n\to\infty} \xi_n(\omega) = \xi(\omega)$ for every $\omega \in \bar{A}$. In this case, we note $\lim_{n\to\infty} \xi_n = \xi$ ($g_\lambda - \boldsymbol{a.s.}$) or $\xi_n \xrightarrow{g_\lambda-\text{a.s.}} \xi$.

In the following, the strong law of large numbers of g_λ variable is proved, which revises the theorem in [24].

Lemma 1 *Let $\xi_1, \xi_2, \cdots, \xi_n$ be independent g_λ variables. If $E\xi_k < \infty$ and $|\xi_k| \le c$ ($k = 1, 2, \cdots, n$), then for every $\varepsilon > 0$*

$$g_\lambda\left\{\max_{k\le n}|S_k - \theta_\lambda(ES_k)| \ge \varepsilon\right\} \le \frac{(1+\lambda)^{\frac{\sum_{k=1}^{n}\theta_\lambda[D\xi_k]}{\varepsilon^2}} - 1}{\lambda}.$$

Proof We stipulate that $S_n = \sum_{k=1}^{n} \xi_k$. Let

$$A_k = \left\{\max_{j\le k}\left|S_j - \theta_\lambda\left[E\left(S_j\right)\right]\right| < \varepsilon\right\},$$

and

$$B_k = A_{k-1} - A_k = \{|S_1 - \theta_\lambda[E(S_1)]| < \varepsilon, \cdots, |S_{k-1} - \theta_\lambda[E(S_{k-1})]| < \varepsilon, |S_k - \theta_\lambda[E(S_k)]| \ge \varepsilon\}.$$

Then, the sets B_k, $k = 1, 2, \cdots, n$ are disjoint. Let $A_0 = X$. We can see that

$$A_0^c = A_0 - A_n = (A_0 - A_1) \cup (A_1 - A_2) \cup \cdots \cup (A_{n-1} - A_n) = \bigcup_{k=1}^{n} B_k$$

and

$$B_k \subset \{|S_{k-1} - \theta_\lambda[E(S_{k-1})]| < \varepsilon, |S_k - \theta_\lambda[E(S_k)]| \ge \varepsilon\}.$$

Moreover, we have

$$\begin{aligned}
&\int_{B_k} |S_n - \theta_\lambda[E(S_n)]|^2 d\theta_\lambda[F_{g_\lambda}(x)] = \theta_\lambda E|(S_n - \theta_\lambda[E(S_n)])\chi_{B_n}|^2 \\
&\ge \theta_\lambda E|(S_k - \theta_\lambda[E(S_k)])\chi_{B_k}|^2 = \int_{-\infty}^{+\infty} [(S_k - \theta_\lambda[E(S_k)])\chi_{B_k}]^2 d\theta_\lambda[F_{g_\lambda}(x)]) \\
&\ge \varepsilon^2 \int_{B_k} d\theta_\lambda[F_{g_\lambda}(x)] \ge \varepsilon^2 \theta_\lambda g_\lambda(B_k).
\end{aligned}$$

Then,

$$\begin{aligned}
\sum_{k=1}^{n}\theta_\lambda(D\xi_k) = \theta_\lambda(DS_n) &= \sum_{k=1}^{n}\int_{B_k} |S_n - \theta_\lambda[E(S_n)]|^2 d\theta_\lambda\left[F_{g_\lambda}(x)\right] \\
&\ge \varepsilon^2 \sum_{k=1}^{n}\theta_\lambda\left[g_\lambda(B_k)\right] = \varepsilon^2\theta_\lambda\left[g_\lambda\left(\bigcup_{k=1}^{n} B_k\right)\right].
\end{aligned}$$

Thus,

$$g_\lambda\left(A_n^c\right) = g_\lambda\left(\bigcup_{k=1}^{n} B_k\right) \le \frac{(1+\lambda)^{\frac{\sum_{k=1}^{n}\theta_\lambda[D\xi_k]}{\varepsilon^2}} - 1}{\lambda}.$$

That is

$$g_\lambda\left\{\max_{k\le n}|S_k-\theta_\lambda(ES_k)|\ge\varepsilon\right\}\le\frac{(1+\lambda)^{\frac{\sum_{k=1}^{n}\theta_\lambda[D\xi_k]}{\varepsilon^2}}-1}{\lambda}.$$

Lemma 2 [24] *Let $\xi_1,\xi_2,\cdots,\xi_n$ be g_λ variables. Then, the following statements are equivalent:*

$(1)\xi_n\xrightarrow{g_\lambda-a.s.}\xi$;
$(2)\forall\varepsilon>0, g_\lambda\left\{\bigcap_{n=k}^{\infty}\bigcup_{n=k}^{\infty}(|\xi_n-\xi|\ge\varepsilon)\right\}=0$;
$(3)\forall\varepsilon>0, \lim_{k\to\infty}g_\lambda\left\{\bigcup_{n=k}^{\infty}(|\xi_n-\xi|\ge\varepsilon)\right\}=0$.

Lemma 3 *Let $\xi_1,\xi_2,\cdots,\xi_n$ be independent g_λ variables. If $E\xi_k<\infty$, $D\xi_k<\infty$ $(k=1,2,\cdots,n)$ and $\sum_n\theta_\lambda\left[D_{g_\lambda}\left(\frac{\xi_n}{n}\right)\right]<\infty$, then*

$$\sum_{k=1}^{n}\left\{\frac{\xi_k}{k}-\theta_\lambda\left[E\left(\frac{\xi_k}{k}\right)\right]\right\}\xrightarrow{g_\lambda-a.s.}0.$$

Proof Let $\xi_k{}'=\frac{\xi_k}{n}$, $k=1,2,\cdots,n$. Then $\xi_k{}'$ $(k=1,2,\cdots,n)$ are also independent g_λ variables. It follows that $S_n{}'=\sum_{k=1}^{n}\frac{\xi_k}{n}=\frac{1}{n}\sum_{k=1}^{n}\xi_k=\frac{S_n}{n}$ and $\theta_\lambda(ES_n')=\theta_\lambda\left\{E\left[\sum_{k=1}^{n}\left(\frac{\xi_k}{k}\right)\right]\right\}$. We need to prove $S_n{}'-\theta_\lambda(ES_n')\xrightarrow{g_\lambda-a.s.}0$.

Clearly, $\bigcup_k\{|S'_{n+k}-ES'_{n+k}|\ge\varepsilon\}=\bigcup_k\left\{\max_{v\le k}|S'_{n+k}-ES'_{n+k}|\ge\varepsilon\right\}$ is a union of some non-decreasing sequences. By Lemma 1, we have

$$g_\lambda\left\{\bigcup_{k=1}^{\infty}\{|S'_{n+k}-\theta_\lambda(ES'_{n+k})|\ge\varepsilon\}\right\}=\lim_{m\to\infty}g_\lambda\left\{\bigcup_{k=1}^{m}\{|S'_{n+k}-\theta_\lambda(ES'_{n+k})|\ge\varepsilon\}\right\}$$

$$=\lim_{m\to\infty}g_\lambda\left\{\max_{k\le m}|S'_{n+k}-\theta_\lambda(ES'_{n+k})|\ge\varepsilon\right\}\le\lim_{m\to\infty}\frac{(1+\lambda)^{\frac{\sum_{k=1}^{m}\theta_\lambda(D\xi'_{n+k})}{\varepsilon^2}}-1}{\lambda}$$
$$=\frac{(1+\lambda)^{\frac{\sum_{k=n+1}^{\infty}\theta_\lambda(D\xi'_k)}{\varepsilon^2}}-1}{\lambda}.$$

Since $\sum_{k=1}^{\infty}\theta_\lambda[(D\xi'_k)]=\sum_{k=1}^{\infty}\theta_\lambda\left[D\left(\frac{\xi_k}{k}\right)\right]<\infty$, we have $\sum_{k=n+1}^{\infty}\theta_\lambda(D\xi'_k)\to0\quad(n\to\infty)$. Then,

$$g_\lambda\left\{\bigcup_k\{|S'_{n+k}-\theta_\lambda[E(S'_{n+k})]|\ge\varepsilon\}\right\}\to0.$$

Thus, $g_\lambda\left\{\bigcup_{n=k}^{\infty}\{|S'_n-\theta_\lambda[E(S'_n)]|\ge\varepsilon\}\right\}\le g_\lambda\left\{\bigcup_k\{|S'_{n+k}-\theta_\lambda[E(S'_{n+k})]|\ge\varepsilon\}\right\}\to0.$

By Lemma 2, we have $S_n{}'-\theta_\lambda[E(S'_n)]\xrightarrow{g_\lambda-a.s.}0$. That is

$$\sum_{k=1}^{n}\left\{\frac{\xi_k}{k}-\theta_\lambda\left[E\left(\frac{\xi_k}{k}\right)\right]\right\}\xrightarrow{g_\lambda-a.s.}0.$$

Lemma 4 [24] Let $A_1, A_2, \cdots$ be a sequence of sets. If $\sum_{k=1}^{\infty}\theta_\lambda\left[g_\lambda(A_k)\right]<\infty$, then $g_\lambda\left\{\bigcap_{n=1}^{\infty}\bigcup_{k\geq n}A_k\right\}=0$.

Lemma 5 Suppose that $\xi_1, \xi_2, \cdots, \xi_n, \cdots$ are identically distributed g_λ variables whose Sugeno distribution function is $F_{g_\lambda}(x)$ with the same expected value a ($a<\infty$). Let $\xi_k^* = \xi_k\chi_{\{|\xi_k|\leq k\}}(\omega)$, $k = 1, 2, \cdots$. If $\sum_{k=1}^{n}\theta_\lambda\left[g_\lambda\left\{\xi_k^*\neq\xi_k\right\}\right]<\infty$ and $\frac{1}{n}\sum_{k=1}^{n}\xi_k^*-\frac{1}{n}\theta_\lambda\left[E\left(\sum_{k=1}^{n}\xi_k^*\right)\right]\xrightarrow{g_\lambda-a.s.}0$, then

$$\frac{1}{n}\sum_{k=1}^{n}\xi_k-\theta_\lambda[a]\xrightarrow{g_\lambda-a.s.}0.$$

Proof Let $\overline{\xi}_n=\frac{1}{n}\sum_{k=1}^{n}\xi_k$, $\overline{\xi}_n^*=\frac{1}{n}\sum_{k=1}^{n}\xi_k^*$, $E\overline{\xi}_n^*=E\left(\frac{1}{n}\sum_{k=1}^{n}\xi_k^*\right)=\theta_{\lambda^{-1}}\left[\frac{1}{n}\sum_{k=1}^{n}\theta_\lambda\left(E\xi_k^*\right)\right]$ and $E\xi_k = a$. We have

$$\begin{aligned}\left|\overline{\xi}_n-\theta_\lambda(a)\right| &= \left|\overline{\xi}_n-\overline{\xi}_n^*+\overline{\xi}_n^*-\theta_\lambda\left[E\left(\overline{\xi}_n^*\right)\right]+\theta_\lambda\left[E\left(\overline{\xi}_n^*\right)\right]-\theta_\lambda(a)\right|\\ &\leq\left|\overline{\xi}_n-\overline{\xi}_n^*\right|+\left|\overline{\xi}_n^*-\theta_\lambda\left[E\left(\overline{\xi}_n^*\right)\right]\right|+\left|\theta_\lambda\left[E\left(\overline{\xi}_n^*\right)\right]-\theta_\lambda(a)\right| \qquad (1)\end{aligned}$$

Because $\sum_{k=1}^{n}\theta_\lambda\left[g_\lambda\left\{\xi_k^*\neq\xi_k\right\}\right]<\infty$, we conclude that $\left|\overline{\xi}_n-\overline{\xi}_n^*\right|\rightarrow^{g_\lambda-a.s.}0$ from Lemma 4 and Lemma 2.

By the condition of $\frac{1}{n}\sum_{k=1}^{n}\xi_k^*-\frac{1}{n}\theta_\lambda\left[E\left(\sum_{k=1}^{n}\xi_k^*\right)\right]\xrightarrow{g_\lambda-a.s.}0$, we have

$$\left|\overline{\xi}_n^*-\theta_\lambda\left[E\left(\overline{\xi}_n^*\right)\right]\right|\xrightarrow{g_\lambda-a.s.}0.$$

Because $\theta_\lambda[E(\xi_n^*)]=\int_{-n}^{n}xd\theta_\lambda[F_{g_\lambda}(x)]\rightarrow\int_{-\infty}^{\infty}xd\theta_\lambda[F_{g_\lambda}(x)]=\theta_\lambda(a)$ as $n\rightarrow\infty$, then $\theta_\lambda\left[E\left(\overline{\xi}_n^*\right)\right]=\frac{1}{n}\sum_{k=1}^{n}\theta_\lambda\left(E\xi_k^*\right)\rightarrow\frac{1}{n}\sum_{k=1}^{n}\theta_\lambda(a)=\theta_\lambda(a)$ as $n\rightarrow\infty$.

By (1), we have $\frac{1}{n}\sum_{k=1}^{n}\xi_k-\theta_\lambda(a)\rightarrow^{g_\lambda-a.s.}0$ $(n\rightarrow\infty)$.

Lemma 6 [26] Let $x_1, x_2, \cdots$ be sequence of real numbers and $\sum_{k=1}^{n}\frac{x_k}{k}<\infty$; then, we have $\frac{1}{n}\sum_{k=1}^{n}x_k\rightarrow 0$, as $n\rightarrow\infty$.

Theorem 2 *(Strong law of large numbers) Let $\xi_1, \xi_2, \cdots, \xi_n, \cdots$ be independent and identically distributed g_λ variables with the same expected value a ($a<\infty$). Then, we have*

$$\frac{1}{n}\sum_{k=1}^{n}\xi_k-\theta_\lambda(a)\overset{g_\lambda\text{-a.s.}}{\longrightarrow}0.$$

Proof Let $\xi_k^* = \xi_k\chi_{\{|\xi_k|\le k\}}(\omega)$.

Since $E\xi_k<\infty$, we have $\int_{-\infty}^{+\infty}|x|d\theta_\lambda\left[F_{g_\lambda}(x)\right]<\infty$, then $\sum_{k=1}^{\infty}\theta_\lambda\left[E\left(\frac{\xi_{k^{*2}}}{k^2}\right)\right]<\infty$ [28]. Thus,

$$\sum_{k=1}^{\infty}\theta_\lambda\left[D\left(\frac{\xi_k^*}{k}\right)\right]\le\sum_{k=1}^{\infty}\theta_\lambda\left[E\left(\frac{\xi_{k^{*2}}}{k^2}\right)\right]<\infty.$$

By Lemma 3, we have $\sum_{k=1}^{n}\left\{\frac{\xi_k}{k}-\theta_\lambda\left[E\left(\frac{\xi_k}{k}\right)\right]\right\}\overset{g_\lambda\text{-}a.s.}{\longrightarrow}0$. By Lemma 6, we have $\frac{1}{n}\sum_{k=1}^{n}\xi_k^*-\frac{1}{n}\theta_\lambda\left[E\left(\sum_{k=1}^{n}\xi_k^*\right)\right]\overset{g_\lambda\text{-}a.s.}{\longrightarrow}0$. By Lemma 5, we have $\frac{1}{n}\sum_{k=1}^{n}\xi_k-\theta_\lambda(a)\overset{g_\lambda\text{-}a.s.}{\longrightarrow}0$ since $\sum_{k=1}^{\infty}\theta_\lambda\left[g_\lambda\{\xi_k^*\ne\xi_k\}\right]<\infty$.

That proves the theorem.

Dependent-Chance Programming on Sugeno Measure Space

Uncertain Environment, Event, Chance Function, and Principle of Uncertainty

Uncertain environment, event, and chance function are basic concepts in DCP. Let us redefine them in Sugeno decision systems at the beginning of this part.

Definition 11 Let $\boldsymbol{x}$ be a decision vector and $\boldsymbol{\xi}$ be a g_λ vector. Then, the Sugeno constraints represented by

$$g_j(\boldsymbol{x},\boldsymbol{\xi})\le 0,\ \ j=1,2,\cdots,p \tag{2}$$

are called a Sugeno environment.

Definition 12 Let $\boldsymbol{x}$ be a decision vector and $\boldsymbol{\xi}$ be a g_λ vector. Then, a system of Sugeno inequalities

$$h_k(\boldsymbol{x},\boldsymbol{\xi})\le 0, k=1,2,\cdots,q \tag{3}$$

is called a Sugeno event.

Definition 13 Let $\boldsymbol{x}$ be a decision vector and $\boldsymbol{\xi}$ be a g_λ vector. Then, the chance function of an event characterized by (3) is defined as the Sugeno measure of the event, i.e.,

$$f(x)=g_\lambda\{h_k(\boldsymbol{x},\boldsymbol{\xi})\le 0, k=1,2,\cdots,q\}$$

subject to the Sugeno environment (2).

Definition 14 [14] Let $\boldsymbol{x}=(x_1,x_2,\cdots,x_n)$ be a decision vector and $r(\boldsymbol{x})=r(x_1,x_2,\cdots,x_n)$ be an n-dimensional function. Then, the ith decision variable x_i is said to be degenerate if

$$r\left(x_1,\cdots x_{i-1},x_i^{'},x_{i+1},\cdots,x_n\right)=r\left(x_1,\cdots x_{i-1},x_i^{''},x_{i+1},\cdots,x_n\right)$$

for any x_i' and $x_i^{''}$; otherwise, it is nondegenerate. In this case, the set of all nondegenerate decision variables is called nondegenerate set under $r(\boldsymbol{x})$ which can be denoted by $ND[r(\boldsymbol{x})]$.

For example, $r(x_1, x_2, x_3, x_4, x_5, x_6) = x_1 - x_2 + 3x_5$ is a 5-dimensional function. The variables x_1, x_2, x_5 are nondegenerate and x_3, x_4, x_6 are degenerate. So,

$$ND[r(x_1, x_2, x_3, x_4, x_5, x_6)] = \{x_1, x_2, x_5\}.$$

Definition 15 Let $\boldsymbol{x}$ be a decision vector, $\boldsymbol{\xi}$ be a g_λ vector, and E be an event characterized by $h_k(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0,\ k = 1, 2, \cdots, q$ in the Sugeno environment $g_j(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0,\ j = 1, 2, \cdots, p$. If $j \in J$ and $ND[g_j(\boldsymbol{x}, \boldsymbol{\xi})] \cap ND[h_k(\boldsymbol{x}, \boldsymbol{\xi})] \neq \phi$, we write

$$\varepsilon^{**} = ND\left[g_j(\boldsymbol{x}, \boldsymbol{\xi})\right] \cup ND[h_k(\boldsymbol{x}, \boldsymbol{\xi})],\quad k = 1, 2, \cdots, q,\quad j \in J$$

Then the jth constraint $g_j(\boldsymbol{x}, \boldsymbol{\xi})$ is called a dependent constraint of the event E if $ND[g_j(\boldsymbol{x}, \boldsymbol{\xi})] \cap \varepsilon^{**} \neq \phi$; otherwise, it is independent.

Definition 16 Let E be a Sugeno event characterized by $h_k(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0, k = 1, 2, \cdots, q$ in the Sugeno environment $g_j(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0,\ j = 1, 2, \cdots, p$ where $\boldsymbol{x}$ is a decision vector and $\boldsymbol{\xi}$ is a g_λ vector. Then, for each decision $\boldsymbol{x}$ and realization of a g_λ vector ξ, the Sugeno event E is said to be consistent in the Sugeno environment if the following two conditions hold: (1) $h_k(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0\ \ k = 1, 2, \cdots, q$ and (2) $g_j(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0,\ j \in J^*$ where J^* is the index set of all dependent constraints.

Generally, a decision could meet an event if the decision meets both the event itself and the dependent constraints [14]. So, we obtain the following principle of uncertainty in the Sugeno environment which is theoretical basis of DCP on Sugeno measure space.

Principle of Uncertainty The chance of a Sugeno event is the Sugeno measure of the event which is consistent in the Sugeno environment.

Let $\boldsymbol{x}$ be a decision vector and $\boldsymbol{\xi}$ be a g_λ vector. There are m events E_i characterized by $h_{ik}(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0,\ k = 1, 2, \cdots, q_i$ for $i = 1, 2, \cdots, m$ in the Sugeno environment $g_j(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0,\ j = 1, 2, \cdots, p$. According to the principle of uncertainty, the chance function of the ith event E_i in the Sugeno environment is

$$f(x) = g_\lambda \left\{ \begin{array}{r} h_{ik}\,(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0, k = 1, 2, \cdots, q_i \\ g_j(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0, j \in J_i \end{array} \right\}$$

where

$$J_i = \left\{ j \in \{1, 2 \cdots p\} \middle| g_j(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0 \text{ is the dependent constraint of } E_i \right\}$$

for $i = 1, 2, \cdots, m$.

DCP on Sugeno Measure Space

In this part, we extend the DCP to the Sugeno measure space. Therefore, the framework of DCP on the Sugeno measure space is constructed. In order to maximize the chance function of an event subject to a Sugeno environment, we give the following dependent-chance single-objective programming on Sugeno measure space:

$$\begin{cases} \max \ g_\lambda\{h_k(\boldsymbol{x},\boldsymbol{\xi})\le 0, k=1,2,\cdots,q\} \\ s.t. \\ \qquad g_j(\boldsymbol{x},\boldsymbol{\xi})\le 0,\ j=1,2,\cdots,p, \end{cases}$$

where $\boldsymbol{x}$ is a decision vector, $\boldsymbol{\xi}$ is a g_λ vector, $h_k(\boldsymbol{x},\boldsymbol{\xi})\le 0$, $k=1,2,\cdots,q$ represent an event, and $g_j(\boldsymbol{x},\boldsymbol{\xi})\le 0$, $j=1,2,\cdots,p$ represent a Sugeno environment.

In order to maximize multiple chance functions subject to a Sugeno environment, we give the following dependent-chance multi-objective programming on Sugeno measure space which maximizes multiple chance functions in a Sugeno environment:

$$\begin{cases} \max \begin{bmatrix} g_\lambda\{h_{1k}(\boldsymbol{x},\boldsymbol{\xi})\le 0, k=1,2,\cdots,q_1\} \\ g_\lambda\{h_{2k}(\boldsymbol{x},\boldsymbol{\xi})\le 0, k=1,2,\cdots,q_2\} \\ \cdots \\ g_\lambda\{h_{mk}(\boldsymbol{x},\boldsymbol{\xi})\le 0, k=1,2,\cdots,q_m\} \end{bmatrix} \\ s.t. \\ \qquad g_j(\boldsymbol{x},\boldsymbol{\xi})\le 0,\ j=1,2,\cdots,p, \end{cases}$$

where $\boldsymbol{x}$ is a decision vector, ξ is a g_λ vector, $h_{ik}(\boldsymbol{x},\boldsymbol{\xi})\le 0$, $k=1,2,\cdots,q_i$ for $i=1,2,\cdots,m$ represent the events, and $g_j(\boldsymbol{x},\boldsymbol{\xi})\le 0$, $j=1,2,\cdots,p$ represent a Sugeno environment.

In multi-objective decision-making system, goal programming is posed to minimize the deviations, positive, negative, or both, between the objective functions and ideal objective targets, which are present in a certain prior structure set by the decision maker [11]. Furthermore, dependent-chance goal programming on Sugeno measure space may be considered as an extension of goal programming in Sugeno decision system. Then, we give the following dependent-chance goal programming on Sugeno measure space:

$$\begin{cases} \min \quad \sum_{j=1}^{l} P_j \sum_{i=1}^{m} \left(u_{ij} d_i^+ + v_{ij} d_i^-\right) \\ s.t. \\ \qquad g_\lambda\{h_{ik}(\boldsymbol{x},\boldsymbol{\xi})\le 0,\ \ k=1,2,\cdots,q_i\} + d_i^+ - d_i^- = b_i,\ \ i=1,2,\cdots,m \\ \qquad g_j(\boldsymbol{x},\boldsymbol{\xi})\le 0,\ j=1,2,\cdots,p \\ \qquad d_i^+, d_i^- \ge 0,\ i=1,2,\cdots,m, \end{cases}$$

where $\boldsymbol{x}$ is a decision vector, $\boldsymbol{\xi}$ is a g_λ vector, P_j is the preemptive priority factor which expresses the relative importance of various goals, for all j, u_{ij}, and v_{ij} are the weighting factors corresponding to positive deviation and negative deviation for goal i with priority j assigned, respectively,

$$d_i^+ = \min\left\{g_\lambda\{h_{ik}(\boldsymbol{x},\boldsymbol{\xi})\le 0,\ \ k=1,2,\cdots,q_i\} - b_i, 0\right\}, i=1,2,\cdots,m$$

and

$$d_i^- = \min\left\{b_i - g_\lambda\{h_{ik}(\boldsymbol{x},\boldsymbol{\xi})\le 0,\ \ k=1,2,\cdots,q_i\}, 0\right\}, i=1,2,\cdots,m$$

are the positive and negative deviations of the target of goal i, respectively, g_j is a function in system constraints, b_i is the target value according to goal i, l is the number of priorities, m is the number of goal constraints, and p is the number of system constraints.

Example 3 Now, we give a simple example of the DCP on Sugeno measure space:

$$\begin{cases} \max & g_\lambda\{x_1 + x_2 = 4\} \\ s.t. & \\ & (x_1 + 3x_2)/4 \le \xi \\ & 2x_3 + x_4 \ge 20 \\ & x_1, x_2, x_3, x_4 \text{ are positive integers,} \end{cases}$$

where ξ is a discrete g_λ variable with the Sugeno distribution of the form in Table 1: and $\lambda = 2$.

In this model, the event E is characterized by $x_1 + x_2 = 4$. And the dependent constraints of the event E are (1) $(x_1 + 3x_2)/4 \le \xi$ and (2) x_1, x_2 are positive integers. By principle of uncertainty, the chance function is

$$g_\lambda \left\{ \begin{array}{c} x_1 + x_2 = 4 \\ x_1 + 3x_2 \le 4\xi \\ x_1, x_2 \text{arepositive integers} \end{array} \right\}.$$

Obviously, (x_1, x_2) can be $(1, 3)$, $(2, 2)$ or $(3, 1)$. According to the different values of (x_1, x_2), we should compare the values:

$$g_\lambda\{\xi \ge 2.5\} = g_\lambda\{\xi = 3\} = 1/2,$$

$$g_\lambda\{\xi \ge 2\} = g_\lambda\{\xi = 3\} = 1/2,$$

and

$$\begin{aligned} & g_\lambda\{\xi \ge 1.25\} \\ & \quad = g_\lambda(\{\xi = 3\} \cup \{\xi = 7/4\}) \\ & \quad = g_\lambda\{\xi = 3\} + g_\lambda\{\xi = 7/4\} + \lambda \cdot g_\lambda\{\xi = 3\} \cdot g_\lambda\{\xi = 7/4\} \\ & \quad = 3/4. \end{aligned}$$

Therefore, the best solution is $(x_1, x_2) = (3, 1)$ with the corresponding chance is 3/4.

A Hybrid Approach to Solve the DCP on Sugeno Measure Space

Sugeno Simulation

In order to estimate the accurate values in Sugeno programming, we resort to Sugeno simulation as one of the attractive alternatives. For the sake of solving general DCP on Sugeno measure space, we must deal with the following type of uncertain function

$$U(\boldsymbol{x}) : \boldsymbol{x} \to g_\lambda\{f(\boldsymbol{x}, \boldsymbol{\xi}) \le 0\}$$

by Sugeno simulation.

Example 4 Let $\boldsymbol{\xi}$ be a g_λ vector and $f: \mathfrak{R}^n \to \mathfrak{R}$ be a measurable function. In the following, we obtain $L = g_\lambda\{f(\boldsymbol{x}, \boldsymbol{\xi}) \le 0\}$ by Sugeno simulation.

In order to construct a g_λ variable ξ with Sugeno distribution $F_{g_\lambda}(\cdot)$, a uniformly distributed variable u over the interval $[0, 1]$ is produced at first, then ξ is assigned to be ${F_{g_\lambda}}^{-1}(u)$ [24]. Therefore, we generate ω_k according to the Sugeno measure g_λ and produce $\boldsymbol{\xi}_k = \boldsymbol{\xi}(\omega_k)$ for $k = 1, 2, \cdots, N$.

Table 1 The Sugeno distribution of ξ

x	1/2	7/4	3
$g_\lambda(\xi = x)$	1/10	1/8	1/2

Let N' denote the number of vectors satisfying the system of inequalities $f(\boldsymbol{x}, \boldsymbol{\xi}_k) \le 0$, $k = 1, 2, \cdots, N$, and

$$h(\boldsymbol{x}, \boldsymbol{\xi}_k) = \begin{cases} 1, & \text{if } f(\boldsymbol{x}, \boldsymbol{\xi}_k) \le 0 \\ 0, & \text{otherwise.} \end{cases}$$

Then, we have $E[h(x, \xi_k)] = L$ for all k and $N' = \sum_{k=1}^{N} h(\boldsymbol{x}, \boldsymbol{\xi}_k)$.

It follows from the Theorem 2 that when $N \to \infty$,

$$\frac{N'}{N} = \frac{\sum_{k=1}^{N} h(x, \xi_k)}{N} \xrightarrow{g_\lambda - a.s.} \frac{(1+\lambda)^L - 1}{\lambda}.$$

Thus, L can be estimated by $\ln[1 + \lambda(N'/N)]/\ln(1 + \lambda)$ provided that N is sufficiently large.

Algorithm 1.

Step 1. Set $N' = 0$.

Step 2. Generate ω_k according to the Sugeno measure g_λ. Equivalently, we generate a g_λ vector $\xi(\omega_k)$ according to its Sugeno distribution.

Step 3. If $f(\boldsymbol{x}, \boldsymbol{\xi}) \le 0$, then $N'{+}{+}$.

Step 4. Repeat the sequence of steps 2-3 N times.

Step 5. Return $L = \ln[1 + \lambda(N'/N)] / \ln(1+\lambda)$.

A Hybrid Approach

In order to solve the DCP on Sugeno measure space, we propose a Sugeno simulation-based hybrid approach which combines BP neural network with GA in this part. The form of the DCP on Sugeno measure space is as follows:

$$\begin{cases} \max & g_\lambda\{h_k(\boldsymbol{x}, \boldsymbol{\xi}) \le 0, k = 1, 2, \cdots, q\} \\ s.t. & \\ & g_j(\boldsymbol{x}, \boldsymbol{\xi}) \le 0, j = 1, 2, \cdots, p. \end{cases}$$

Firstly, we generate the input-output data for the uncertain function according to the Sugeno simulation. Secondly, we train the BP neural network to approximate the underlying functional relationship U and predict the outputs. Thirdly, we make use of GA to enhance the optimization process and arrive at a solution to the optimization problem. Finally, we find the best chromosome to be the optimal solution by selection, crossover, and mutation.

The hybrid algorithm for solving the DCP on Sugeno measure space can be summarized as follows:

Algorithm 2.

Step 1. Give system modeling. And generate input–output data for the following uncertain function

$$U : \boldsymbol{x} \to g_\lambda \left\{ h_k \left(\boldsymbol{x}, \boldsymbol{\xi} \right) \le 0, k = 1, 2, \quad , q; g_j \left(\boldsymbol{x}, \boldsymbol{\xi} \right) \le 0, j \in J^* \right\}$$

by Sugeno simulation.

Step 2. Construct a BP neural network to approximate the functions according to the generating input-output data.

Step 3. Initialize and train the BP neural network.

Step 4. Initialize a certain number of chromosomes according to the Sugeno distribution function.

Step 5. Calculate the values of the objective function as fitness value by the trained BP neural network.

Step 6. Select the chromosomes by running a standard scheme of the roulette wheel.

Step 7. Breed new feasible candidate chromosomes by crossover operation and further modified through mutation operation.

Step 8. Evaluate the values of the objective function for all chromosomes by the trained BP neural network, and then choose the chromosome whose fitness is maximal according to the objective function values.

Step 9. Repeat Step 6 to Step 8 for a given number of cycles.

Step 10. Report the best chromosome which is regarded as a solution.

Numerical Examples

Here, we give two numerical examples to illustrate the effectiveness of the approach.

Example 5 Let us consider the following DCP on Sugeno measure space:

$$\begin{cases} \max & g_\lambda\{x_1 + x_2 + 2x_3 + 2x_4 = 5\} \\ s.t. & \\ & {x_1}^2 + {x_2}^2 \le \xi_1 \\ & {x_3}^2 + {x_4}^2 \le 3\xi_2 \\ & x_1, x_2, x_3, x_4 \ge 0, \end{cases}$$

where $\lambda = 5$; ξ_1 is a Sugeno normal distributed variable characterized by $\xi_1 \sim SN(3, 2^2, 4)$; ξ_2 is a λ - 0 - 1 distributed variable characterized by $\xi_2 \sim SU(3)$.

The event of this model is $x_1 + x_2 + 2x_3 + 2x_4 = 5$, and the chance function of the event is

$$f(x) = g_\lambda \left\{ \begin{array}{c} x_1 + x_2 + 2x_3 + 2x_4 = 5 \\ {x_1}^2 + {x_2}^2 \le \xi_1 \\ {x_3}^2 + {x_4}^2 \le 3\xi_2 \\ x_1, x_2, x_3, x_4 \ge 0 \end{array} \right\}.$$

It is easily to find that $x_4 = \frac{5 - x_1 - x_2 - 2x_3}{2}$. In order to solve the model, we generate 3000 input-output data for the uncertain function $U : (x_1, x_2, x_3) \to f(x)$ by Sugeno simulation. Next, a three-layer BP neural network of the 3-5-1 topology is constructed to approximate the uncertain function U. The activation functions utilized in the BP neural network are a hyperbolic tangent function (hidden layer) and a linear function (output layer), respectively. The BP neural network is trained to adjust the weights and thresholds of the connections between two layers and minimize root mean squared error (RMSE) of the output layer. The maximum number of iterations, the learning rate, the momentum term, and the tolerance criterion for the BP neural network are set to be 5000, 0.0001, 0.85, and 0.001, respectively. Although the settings of these parameters may not be optimal, they ensure the convergence of the learning process realized by the BP neural network. As shown in Fig. 1, we obtain the values of the error function of RMSE in successive iterations.

Moreover, we use the GA to improve the solution of the DCP on Sugeno measure space. Here, the population size, the number of generations, the mutation rate, and the crossover rate of the GA are set to be 30, 300, 0.2, and 0.3, respectively. It should be mentioned here that the settings of these GA parameters may not be optimal. However, under these conditions, the value of fitness (the objective function) is improved. The improvements are illustrated in Fig. 2.

Finally, the best solution obtained in the above way is

$$x^* = (x_1, x_2, x_3,\ x_4) = (1.1379, 0.7077, 0.7579, 0.8183)$$

where the corresponding chance is $f(x^*) = 0.9067$.

Example 6 Let us consider the following dependent-chance goal programming on Sugeno measure space:

$$\begin{cases} \text{lexmin}\{d_1^+, d_2^+, d_3^+\} \\ s.t. \\ \quad g_\lambda\{x_1 + {x_4}^2 = 2\} + d_1^- - d_1^+ = 0.80 \\ \quad g_\lambda\{x_2 + {x_5}^2 = 2\} + d_2^- - d_2^+ = 0.85 \\ \quad g_\lambda\{x_3 + {x_6}^2 = 3\} + d_3^- - d_3^+ = 0.85 \\ \quad x_1^2 + x_5 + x_4^2 \le 0.5\xi_1 \\ \quad x_3 + x_2^2 + x_6 \le 2.5\xi_2 \\ \quad x_i \ge 0, i = 1, 2, \cdots 6 \\ \quad d_i^+, d_i^- \ge 0, i = 1, 2, \cdots 6, \end{cases}$$

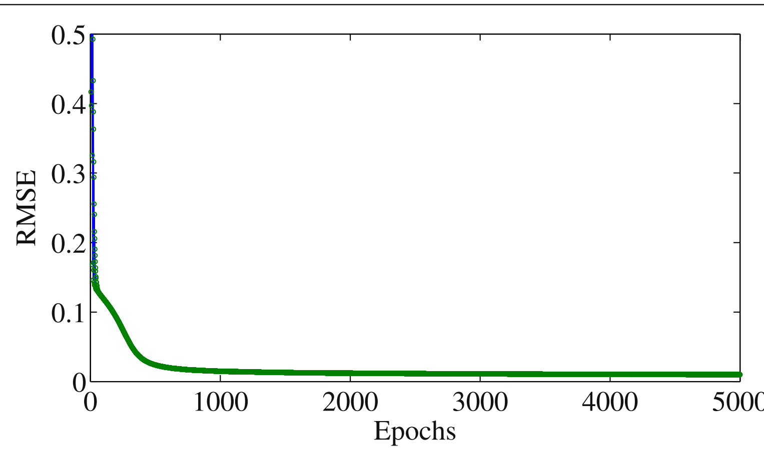

Fig. 1 Training error curves for $f(x)$ obtained for 3000 input–output data

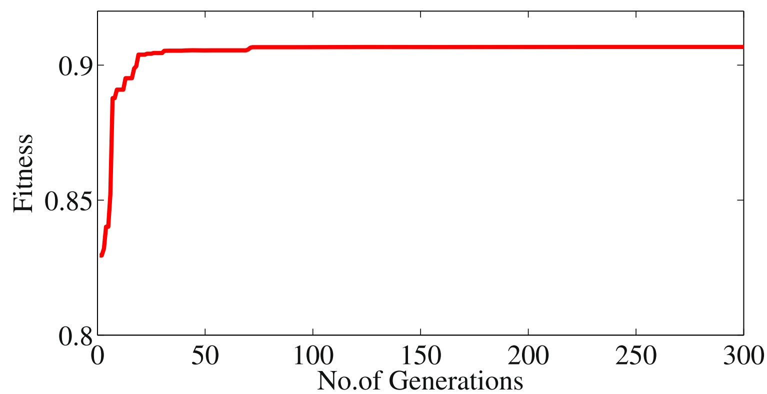

Fig. 2 Values of objective function for 300 generations of the GA

where $\lambda = 3$; ξ_1 and ξ_2 are g_λ variables characterized by $\xi_1 \sim SN(3,1,2)$ and $\xi_2 \sim SN(2,1,3)$, respectively.

The event at the first priority level is $x_1 + x_4^2 = 3$ whose chance function is

$$f_1(x) = g_\lambda \left\{ \begin{array}{l} x_1 + {x_4}^2 = 2 \\ x_1^2 + x_5 + x_4^2 \leq 0.5\xi_1 \\ x_1, x_4, x_5 \geq 0 \end{array} \right\}.$$

The event at the second priority level is $x_2 + x_5^2 = 2$ whose chance function is

$$f_2(x) = g_\lambda \left\{ \begin{array}{l} x_2 + x_5^2 = 2 \\ x_1^2 + x_5 + x_4^2 \leq 0.5\xi_1 \\ x_3 + x_2^2 + x_6 \leq 2.5\xi_2 \\ x_i \geq 0, i = 1, 2, \cdots 6 \end{array} \right\}.$$

The event at the third priority level is $x_3 + x_6^2 = 2$ whose chance function is

$$f_3(x) = g_\lambda \left\{ \begin{array}{l} x_3 + {x_6}^2 = 3 \\ x_3 + x_2^2 + x_6 \leq 2.5\xi_2 \\ x_2, x_3, x_6 \geq 0 \end{array} \right\}.$$

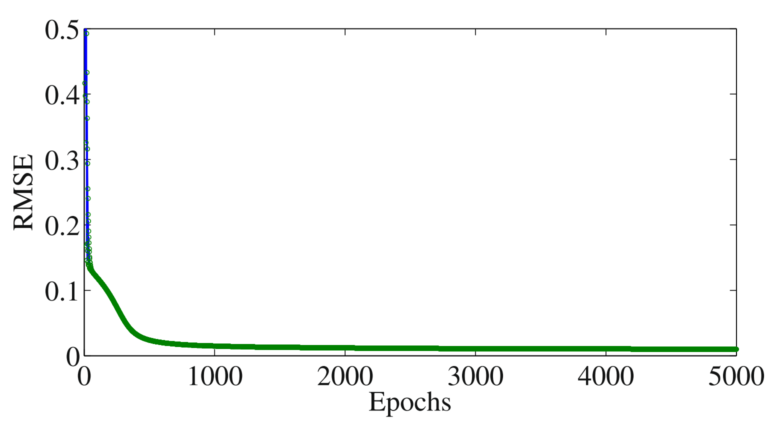

Fig. 3 Training error curves for $f_1(x)$ obtained for 3000 input–output data

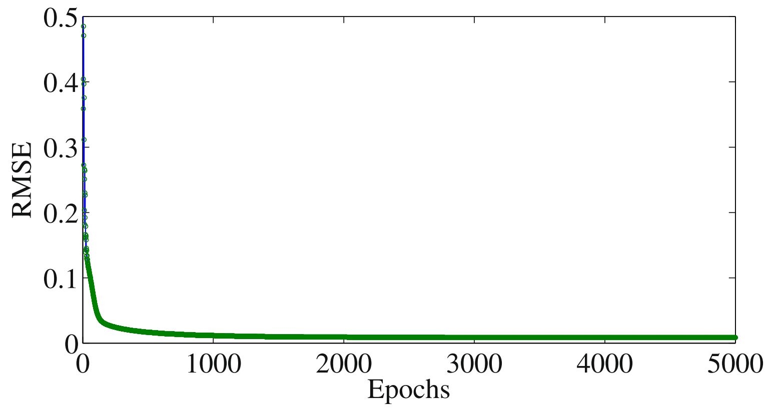

Fig. 4 Training error curves for $f_2(x)$ obtained for 3000 input–output data

We can find that $x_4 = \sqrt{2-x_1}$, $x_5 = \sqrt{2-x_2}$ and $x_6 = \sqrt{3-x_3}$. In order to solve this model, we generate 3000 input-output data for the uncertain function $U:(x_1,\ x_2,\ x_3) \to (f_1(x),\ f_2(x),\ f_3(x))$ by Sugeno simulation. Next, we construct a three-layer BP neural network of the 3-5-3 topology to approximate the uncertain function U. The BP neural network is trained by the standard BP algorithm with a momentum term while the error function is RMSE. The maximum number of iterations, the learning rate, the momentum term, and the tolerance criterion for the BP neural network are set to be 5000, 0.0001, 0.85, and 0.001, respectively. The values of the error functions obtained in successive iterations are shown in Figs. 3, 4 and 5, respectively.

Then, we use the GA to improve the solution of the DCP on Sugeno measure space, whose population size, number of generations, mutation rate, and crossover rate are set to be 30, 300, 0.2, and 0.7, respectively. The GA enhances the fitness as shown in the Fig. 6,

Finally, the optimal solution is

$$x^* = (1.0274, 2.000, 0.0001, 0.9863, 0, 1.7320),$$

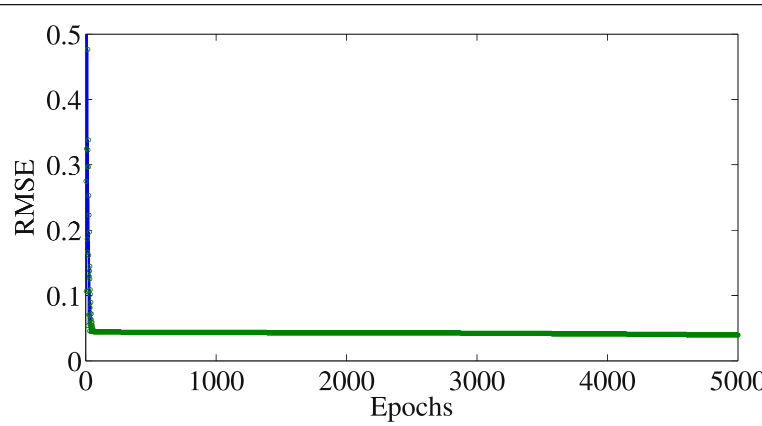

Fig. 5 Training error curves for $f_3(x)$ obtained for 3000 input–output data

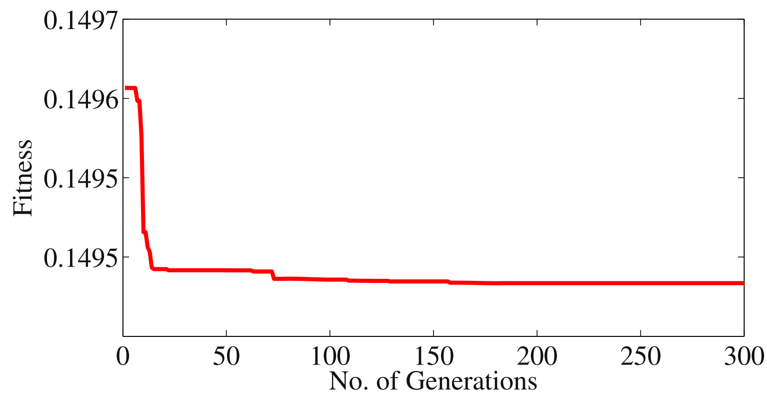

Fig. 6 Values of objective function for 300 generations of the GA

which satisfies the first goal and the second goal; otherwise, the third objective is 0.1495.

In the process of solving the above two models, we can see that the time complexity of the hybrid approach is the sum of the time spent for the Sugeno simulation, for BP neural network, and for GA. The time spent for the three parts are essential since we can assumed that there is no alternative method to the hybrid approach.

Conclusions

In this paper, an uncertain mathematical programming named dependent-chance programming (DCP) on Sugeno measure space was proposed. To provide general solutions to the programming, a Sugeno simulation-based hybrid approach integrated by BP neural network and GA was given. Compared with the existing kinds of DCP, the DCP on Sugeno measure space has the features as follows: (1) It deals with g_λ variables. (2) It may be resorted to when the decision maker wishes to maximize the chance functions of satisfying the events in the Sugeno environment.

Further research directions might be devoted to the wide applications of DCP on Sugeno measure space in area of water resources management, waste management planning, and electric power system planning, and so on, where some characteristics may not satisfy the additivity. Moreover, the DCP based on other kinds of variables, such as fuzzy variables, on Sugeno measure space may be studied. And the hybrid approach for DCP combined with more algorithms such as PSO may be also studied.

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Authors' contributions

All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no conflict of interests.

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